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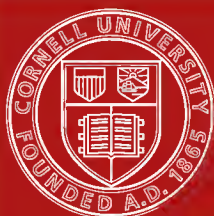
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A LABORATORY COURSE
IN
EXPERIMENTAL PHYSICS



A LABORATORY COURSE

EXPERIMENTAL PHYSICS

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PREFACE.



AT the present day, when students are required to gain knowledge of natural phenomena by performing experiments for themselves in laboratories, every teacher finds that as his classes increase in number, some difficulty is experienced in providing, during a limited time, ample instruction in the matter of details and methods.

During the past few years we ourselves have had such difficulties with large classes; and that is our reason for the appearance of the present work, which is the natural outcome of our experience. We know that it will be of service to our own students, and hope that it will be appreciated by those engaged in teaching Experimental Physics elsewhere.

The book contains a series of elementary experiments specially adapted for students who have had but little acquaintance with higher mathematical methods: these are arranged as far as possible in order of difficulty. There is also an advanced course of experimental work in Acoustics, Heat, and Electricity and Magnetism, which is intended for those who have taken the elementary course.

The experiments in Acoustics are simple, and of such a nature that the most of them can be performed by beginners in the study of Physics; those in Heat, although not requiring more than an ordinary acquaintance with Arithmetic, are more tedious and apt to test the patience of the experimenter; while

the course in Electricity and Magnetism has been arranged to illustrate the fundamental laws of the mathematical theory, and involves a good working knowledge of the Calculus.

The important subject of Physical Optics has been omitted because we have been unable to map out a course of experiments which would be in accordance with the ordinarily accepted theory of Fresnel. At an early date, however, we hope to place before the public a connected series of experiments (based upon true fundamental laws) in that most interesting of all physical studies.

A short appendix is given on the methods of determining the value of Gravity and on the use of the Torsion Pendulum.

Finally, we may say that throughout the work simplification of method has been our greatest, if not our only, aim; for therein we believe lies the only true sign of Progress.

UNIVERSITY OF TORONTO, April, 1895.

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PART I.

ELEMENTARY COURSE

A LABORATORY COURSE IN EXPERIMENTAL PHYSICS.



EXPERIMENTS.

I. THE VERNIER.

To measuring instruments provided with a scale there is frequently attached a more finely divided one, by means of which readings can be made with greater precision. This latter scale is called a *vernier*, and is arranged so as to slide along the scale proper, either edge to edge, or else one overlapping the other. In this latter case the edge of the upper scale is beveled. Generally a length equal to $n - 1$ divisions on the scale is taken on the vernier, and this length divided into n equal parts, so that one of the vernier divisions is equal to $n - 1/n$ of a scale division. In this way, if the zero line on the vernier coincides with a line on the scale, then the space between line 1 on the vernier, and the line next to it on the scale is $1/n$ of a scale division, that between line 2 on the vernier, and the line next to it on the scale is equal to $2/n$ of a scale division, while that between the r th line on the vernier, and the line next to it on the scale is equal to r/n of a scale division, and the n th line on the vernier will, just as the zero line, coincide with a line on the

scale. By following out a process, the converse of that just described, it will be readily seen that if the r th line on the vernier coincides with a line on the scale, then the space between the vernier zero, and the line next behind it on the scale is r/n of a scale division.

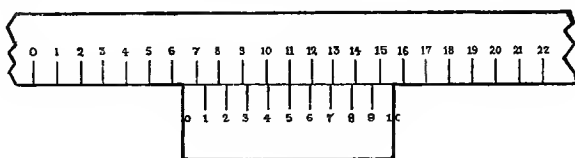


Fig. 1.

Different values are given to n , according to the purpose for which the vernier is to be used. Figure 1 is an example of a vernier in which n is equal to 10. In this case, line 5 on the vernier coincides with a line on the scale, so that the space between the vernier zero, and the line on the scale next behind it is $\frac{5}{10}$ of a scale division. The reading indicated is therefore 6.5.

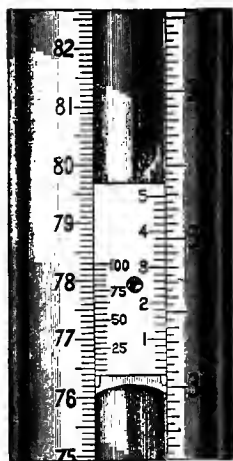


Fig. 2.

In the vernier on the right-hand side of Fig. 2, n is taken equal to 25. Line 18 on the vernier coincides with a line on the scale, and the vernier zero is therefore $\frac{18}{25}$ of a scale division above the line next behind it on the scale. Here one of the small scale divisions is equal to .05 of a large one, and therefore the space mentioned above will be equal to .036 of a large scale division, and the reading is $30.05 + .036$, or 30.086.

In Fig. 3, *A*, *B*, *C*, and *D* are examples of this class of vernier, and it will be an instructive exercise for the student to verify the readings there given. In order to avoid the calculation just indicated, verniers are often so numbered that readings

can be made directly. One of this class is exhibited on the left-hand side of Fig. 2. There n is equal to 20, and since line 19 on the vernier coincides with a line on the scale there is there-

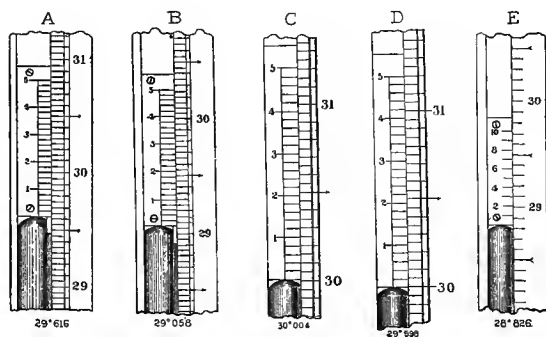


Fig. 3.

fore $\frac{19}{20}$, or .95, of a scale division between the vernier zero and the line next behind it on the scale. From the manner in which the spaces are numbered, it can be readily seen that line 19 is

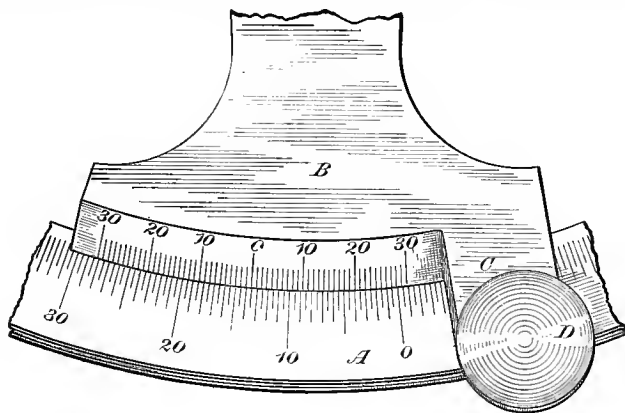


Fig. 4.

represented by 95, and the reading 76.395 may thus be made directly. It frequently happens, when a small value is given to n , that no line on the vernier coincides with one on the scale.

An example of this is shewn in *E* of Fig. 3, where an approximation has been made.

Figure 4 is an example of a circular vernier. In this case, n is equal to 30, and since the small scale divisions are each equal to one-half a degree readings can be taken to minutes. The reading indicated is $13^{\circ} 37'$. Besides those already described, verniers are sometimes constructed in which $n - 1$ of their divisions are equal to n scale divisions. However, in this case no special difficulty will be met with, as the same principles apply to all classes of verniers.

II. THE CALIPERS.

This instrument, shewn in Fig. 5, is essentially a graduated metal scale, to one end of which (and perpendicular to it) is attached a bar *EP*. Another bar, *FQ*, of the same form, is capa-

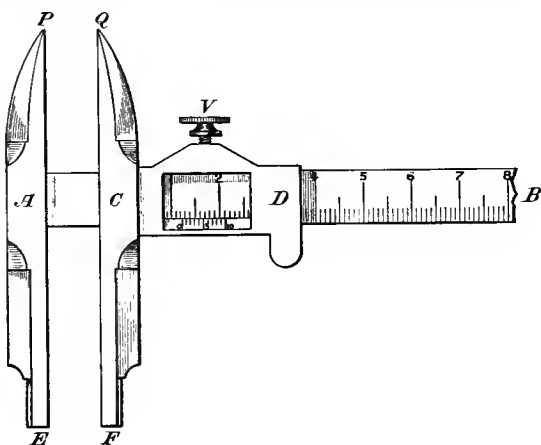


Fig. 5.

ble of being moved along *AB*, and carries with it a slider *CD*, which can be fixed by the screw *V* at any point on *AB*. There is a small rectangular opening in *CD*, and on one of its bevelled edges a vernier is ruled, which is so placed that when the bars *EP* and *FQ* are together, the zero on the vernier corresponds

with the zero on the scale. In measuring the length of an object, place it between the branches PA and QC , and gently press the slider against it. Then note the reading on the vernier.

The calipers may also be used to measure the width of an opening in a tube, or other object, provided it is greater than the total thickness of EP and FQ when they are together. In this case, insert AE and CF in the opening, and separate them as far as its walls will permit. To the vernier reading then must be added the thickness of AE and CF when they are

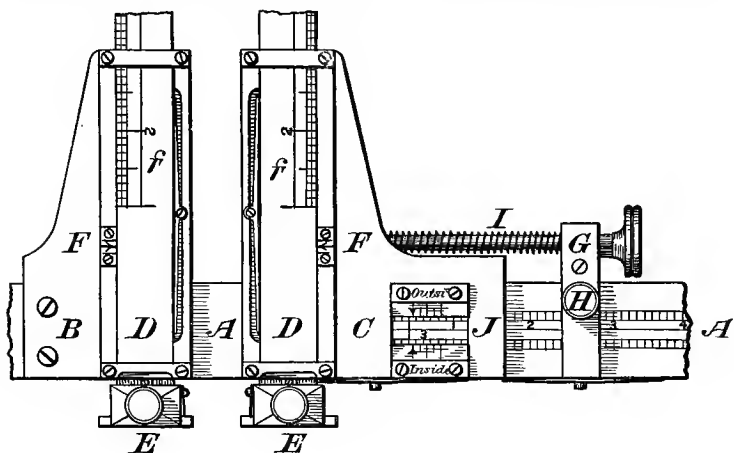


Fig. 6.

together, which may be found by means of another instrument. It will be a useful exercise for the student to test, at various times, the accuracy of the calipers, by means of a standard instrument, one of which is shewn in Fig. 6.

In this standard the object to be measured is placed between the two heads E, E of the branches D, D . These are free to slide in the sockets F, F , so that measurements may be taken between points not accessible by instruments with fixed branches. If it is desired to find the width of an opening, the two heads E, E are inserted in it, and then separated as far as

possible. By means of a special scale and vernier on the arm *AB*, readings for such measurements can be taken directly.

III. THE CATHETOMETER.

The cathetometer is used to measure the vertical distance between two horizontal planes passing through two points not in general in the same azimuth. As exhibited in Fig. 7, it consists

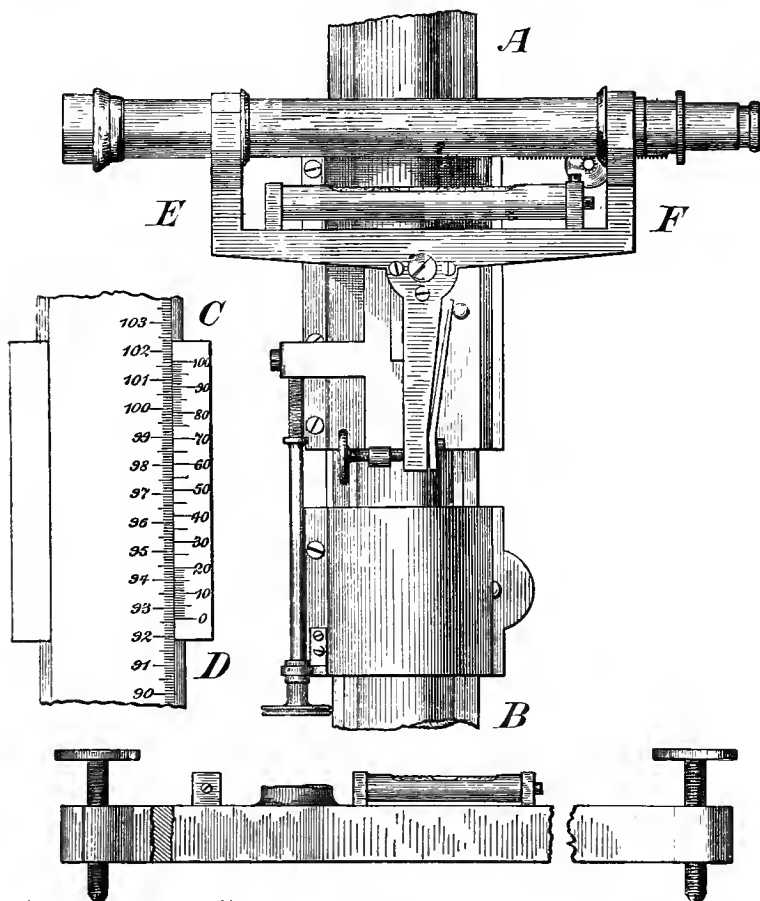


Fig. 7.

of a vertical metal cylinder AB with a scale CD inserted in it, is movable about a vertical axis, and is supported on a platform furnished with three adjustment screws. On this platform are two spirit levels, by means of which the perpendicularity of AB may be tested. Attached to this vertical column, or cylinder, is a carriage EF , upon which is placed a telescope with its optical axis horizontal. This carriage is capable of sliding up and down the column, and can be fixed at any point by means of a compression screw. It also has a vernier attached to it, and carries a spirit level to adjust the telescope.

To make a measurement, sight the telescope on one of the points so that its image coincides with the intersection of the crosswires. Note the reading on the vernier, and then move the carriage, and the telescope up or down, as the case requires, until the image of the other point coincides with the intersection of the crosswires. By means of the reading then made, and the previous one, the vertical distance between the points may be found. The instrument is chiefly used in finding the heights of columns of mercury, examples of which are found in the experiment on the *capillary constant*, and in that on the *absolute expansion of mercury*.

If the two points are at a considerable distance from the cathetometer, it will be found that better results may be obtained by placing a scale, in a vertical position, alongside of these points, and then, by means of the telescope on the instrument, noting what division on the scale corresponds with each of the points respectively. In the construction of the instrument, it is very difficult to make the column AB perfectly rigid, and to keep it so; and from this cause the readings are often not as accurate as is desirable. The instrument may be tested, from time to time, by measuring the spaces between lines, drawn with a dividing engine.

IV. THE SPHEROMETER.

The essential part of the spherometer (Fig. 8) is a micrometer screw DP , which passes through a metal tripod whose feet are of equal length. This screw carries with it, at its upper end,

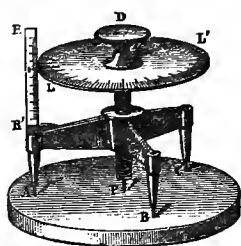


Fig. 8.

a graduated metallic circular disc LL' , whose plane is perpendicular to the screw, and parallel to the plane passing through the three terminals A, B, C , of the supports. To one of these supports there is attached a scale RR' graduated in such a manner that on causing the disc to make one complete revolution, it will rise or fall through one of these divisions.

Usually the scale is divided into spaces one-half a millimeter in width, and the disc into 500 equal parts so that readings can be taken to $\frac{1}{1000}$ of a millimeter when any particular measurement is about to be made. It is first of all necessary to note the reading when the point P of the micrometer screw is in the same plane as the three terminals A, B, C , and the instrument is resting equally on the four supports. Usually to the beginner this presents some difficulty; but with a little practice, and by repeatedly grasping one of the supports A, B , and C , and giving the instrument a slight rotatory movement, one can easily perceive when contact has been made with the screw. If the instrument is standing on a glass, or any reflecting surface, it can easily be ascertained when contact has been made by noting the instant when the point P coincides with its image as seen in the surface. The instrument can be used, (1) to measure small thicknesses, (2) to determine the radius of curvature of a sphere, and (3) to test whether a surface is perfectly plane, or perfectly spherical. If it is required to measure the thickness of a plate with parallel faces, first place the instrument on a perfectly plane surface (usually made of ground glass), and turn the screw until its point P comes in

contact with it; note the reading, and then raise the screw and place under the point P the plate whose thickness is to be determined. Bring the point of the screw in contact with the upper surface of this plate, and again note the reading. The difference between these two readings will give its thickness. It is evident that if the instrument be equally supported on the four points A , B , C , and P in one position, and then caused to glide over a given surface, it can be readily seen whether the surface is perfectly plane, or not.

To find the radius of curvature of a spherical surface, place the spherometer on it, and adjust the micrometer screw until the instrument rests equally on the four supports. Note the reading, and then place the spherometer on a plane surface, and again establish contact at four points, and note the reading. The difference between these two will give the height h of the spherical segment which has for its base a plane passing through the terminals A , B , and C , when the first reading was taken.

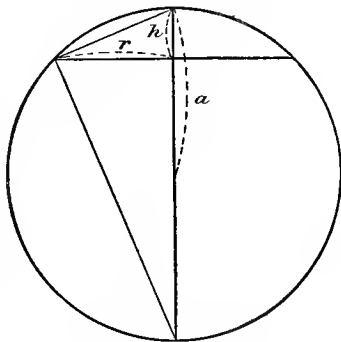


Fig. 9.

If then r be the radius of the base of this segment, h its height, and a the radius of the sphere, we have from Fig. 9,

$$r^2 = h(2a - h),$$

$$a = \frac{r^2}{2h} + \frac{h}{2}.$$

To find r , place the instrument on a sheet of cardboard resting on the three supports, and the screw, and press it lightly down so as to make small indentations. Since the three points A , B , and C form an equilateral triangle, r may be determined

by taking the mean of the distances from each of the holes made by *A*, *B*, and *C* to that made by the point of the screw.

Whether a surface is perfectly spherical, or not, may be determined by adjusting the instrument to four-point contact in one position, and then sliding it over the surface.

In some instruments the screw is hollow, and throughout its length a slender metallic rod runs. This rod is connected to a lever, and a pointer, which indicates at once the instant when contact is made.

V. MICROMETER SCREW GAUGE.

This gauge (Fig. 10) is generally used to measure the diameters of fine wires. It is constructed on the same principle as the spherometer, and consists of a bracket or shoulder with two arms *FE* and *FG*, in the former of which is screwed a small metal cylinder *C*, with its protruding face perfectly plane, and

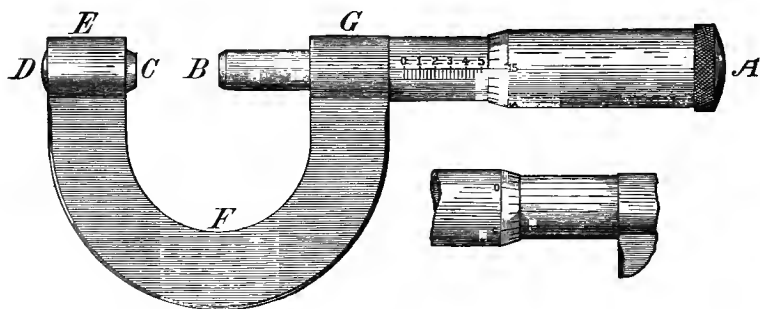


Fig. 10.

at right angles to the axis of the micrometer screw *AB*, which passes through a threaded tube attached to the arm *FG*. The end of the screw *B* is made plane, and is parallel to the face of the cylinder *C*. To the other end there is attached a cap *A*, which, when the screw is turned, comes down over the tube on *FG*. This latter has a scale ruled on it, and the divisions are such that when the screw is made to turn through one complete revolution, the cap *A* moves over one of the divisions. Just as

in the case of the disc of the spherometer, the circumference of the cap is divided into a number of equal parts, and readings may thus be made to the fraction of a division. In some gauges the scale is divided into millimeters, and the circumference of the cap into twenty equal parts, so that readings may be taken to $\frac{1}{20}$ of a millimeter; while in others the scale is divided into spaces of $\frac{1}{40}$ of an inch, and the circumference of the disc into 25 equal parts. In this latter case, readings may be taken to $\frac{1}{1000}$ of an inch. For still finer readings a vernier is sometimes ruled longitudinally on the back of the tube, in the manner shown in the figure.

In making a measurement, first see that, when the screw is in contact with *C*, the zero on the cap corresponds with that on the scale; if not, it may be made to do so by screwing the cylinder *C* either in or out, as the case requires. The wire to be measured is then placed between the face of *C* and that of the screw, and the latter turned gently until it is felt that contact is made. The reading then indicated will give the diameter of the wire. Care should be taken to insure the same manner of making contact while measuring the wire as in testing for the zero reading.

VI. DIVIDING ENGINE.

The dividing engine, of which the essential parts are shewn in Fig. 11, is employed in ruling scales, and gratings, and in measuring small horizontal distances. It embodies the same principle as the spherometer, and consists of a screw *B*, whose pitch, generally one, or one-half a millimeter, is very regular; a divided circle *C* attached to one end of it, which can be so adjusted that a fractional part of a rotation may be given to the screw; and a carriage *EE*, which slides along the smooth rail *I* attached to the solid base *A*. This carriage *EE*, on which are supported a tracing point *F* with a handle *G*, and a microscope *H*, by means of which the rulings are examined, may be thrown in or out of

gear with the screw *B* by turning the rod *K*, which opens the clutch shown in the figure.

The object which is to be ruled, or measured, is placed on

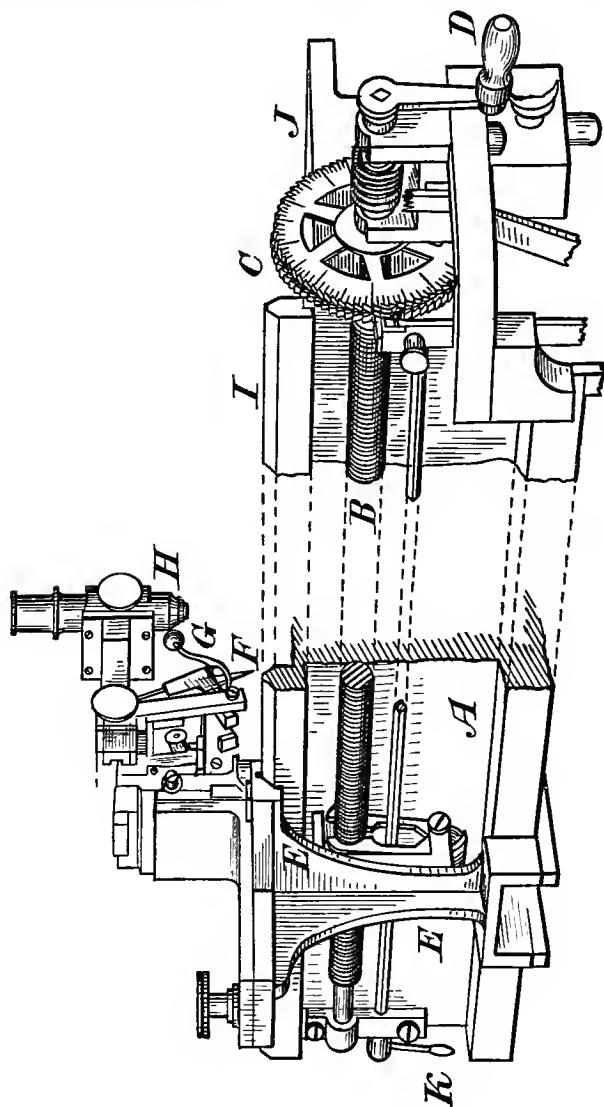


Fig. 11.

the platform J directly beneath the tracing point, and the microscope.

Scales on metal may be ruled either directly by using a hardened steel tracing point, or by ruling on a wax-covered surface, and then etching in with acid. This latter method may also be adopted in the case of glass scales, but these are generally made by using a diamond tracing point. For such fine work as ruling gratings, dividing engines are specially constructed with screws whose pitch depends on the degree of delicacy required in the work.

VII. SPECIFIC GRAVITY BOTTLE.

Figure 12 shows the specific gravity bottle, with ground glass stopper, generally used in laboratory experiments. Throughout the length of the stopper, a very fine hole runs, which is terminated in a small cup-shaped opening at the top. A mark is made on the narrow part of the stopper, and enough liquid is put in the bottle to make it rise to this point when the stopper is in tight. In what follows, the bottle will always be considered full when the surface of the liquid is in this position.



Fig. 12.

Great care should be taken to have the bottle perfectly clean. This is done by first washing it out with a solution of caustic potash, and then with hydrochloric acid. After this, wash it several times with ordinary water, and then rinse out with distilled water, and place it in a hot air bath to dry. In filling the bottle with a liquid, the student should be careful to have all the air bubbles removed.

EXERCISE I. — *To find the specific gravity of a liquid.*

Let w be the weight of the bottle in air, w' its weight when filled with distilled water, and w'' when filled with the given liquid. Equal volumes of the liquid, and of the water weigh

then $w'' - w$, and $w' - w$, respectively, and the specific gravity of the former is therefore given by $\frac{w'' - w}{w' - w}$.

EXERCISE II. — *To find the specific gravity of a solid broken up into small pieces.*

Let w be the weight of the pieces of the solid, w' the weight of the bottle filled with distilled water, and w'' the weight of the bottle partly filled with the pieces, and the balance filled with water. Then $w + w' - w''$ will be the weight of the water displaced by the pieces, and the specific gravity of the solid is therefore

$$\frac{w}{w + w' - w''}$$

If the solid is soluble in water, the same method may be used to find its specific gravity relative to some liquid in which it is not soluble, and then, the specific gravity of this liquid having been determined, that of the solid may be calculated.

A suitable exercise on this method is to find the specific gravity of wires used in the experiments on the sonometer. In this case the wire should be cut up into small pieces unless it is very fine.

If very great accuracy is required in these experiments, corrections will have to be made for temperature, and for the buoyancy of the air.

VIII. HYDROSTATIC BALANCE.

Experiment I. — In Fig. 13 a balance is shewn whose scale pans are at a considerable height above the platform on which it stands. To one of these pans, as shewn in the figure, attach a cylindrical cup A , and also the cylindrical solid B , which just fills this cup. Then balance the two by weights placed in the opposite pan. Having done this, place a beaker of water under A and B , so that B is completely submerged, and shew that on filling A with water equilibrium is restored. This experiment

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water equilibrium is restored. This experiment shews that the reaction of the cylinder *B* on the water, which is equal to the weight of the water it displaces, is equal to the action of the water on the cylinder. By adopting this method, the weight of the water displaced by a given solid may be found directly, and it will be very instructive for the student to use it in finding the specific gravities indicated in Exercises I., II., and III.

IX. NICHOLSON'S HYDROMETER.

The apparatus (Fig. 15) consists of a hollow metallic vessel *B*, to one end of which is attached a slender stem terminated in a pan *D*, capable of holding weights, or the solid whose specific gravity is to be determined. To the other end of the instrument there is attached a second platform, or pan *C*, made very heavy, so that when the instrument is at rest in water it will assume the position indicated in the figure.

When placed in water, the instrument sinks until it is only partially submerged, and by placing weights on the pan *D*, it may be made to sink to any required depth.

To find the specific gravity of a solid insoluble in water.

Let the weight *P*, when put on the pan *D*, sink the instrument to the mark *A* on the stem, in pure distilled water. Remove this weight, and place the given solid on the pan. Let *p* be the weight that must now be added to cause the hydrometer to sink to the mark *A*; $P - p$ will therefore be the weight of the solid. Leave the weight *p* on the pan *D*,

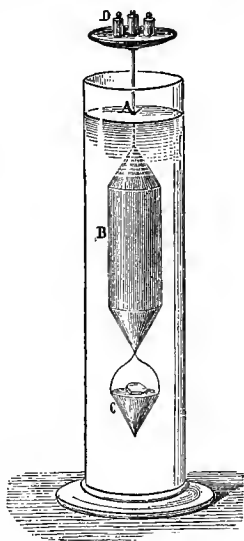


Fig. 15.

and place the solid then on the pan *C*, and since the solid will now lose a part of its weight equal to that of the water it displaces, more weights must be placed on the pan *D* to sink the instrument to the mark *A*. Let P' be the amount of these weights. Then P' will be the weight of the water displaced by the solid; and, since its own weight is $P - p$, its specific gravity is therefore $\frac{P - p}{P'}$.

If the solid used is lighter than water, it will tend to rise when placed on the pan *C*. In this case it is prevented from rising by being placed in a wire cage attached to *C*, and the experiment is conducted exactly in the manner indicated above. If the solid to be tested is soluble in water, the experiment may be conducted by using some liquid of known specific gravity in which it is not soluble.

If the stem of the instrument be made very slender, the hydrometer will be sensitive to small variations in the weights placed on the pan; but owing to the surface tension of the water on this stem, and on the walls of the instrument, the results obtained will not be as accurate as those obtained when either of the two previous methods is adopted. The instrument, however, enables us to find rapidly the specific gravity of a solid approximately. Corrections may also be made in this experiment for the temperature of the water.

X. FAHRENHEIT'S HYDROMETER.

Fahrenheit's hydrometer (Fig. 16) embodies the same principle as that of Nicholson, and resembles it in form, except that it is made of glass so that it may be used in all liquids. To its lower extremity there is attached a small glass bulb generally loaded with mercury or small shot. This causes the instrument to float in an erect position.

To find the specific gravity of a liquid.

Let P be the weight of the instrument when weighed in air, and p the weight which, when placed on the pan, causes it to

sink in pure distilled water to the mark *A* on the stem. Let p' be the weight which must be placed in the pan to cause it to sink to this mark when placed in the given liquid. Then, since $P+p$, and $P+p'$ are the weights of equal volumes of water, and of the given liquid respectively, the specific gravity of the latter is therefore $\frac{P+p'}{P+p}$.

Hydrometers of constant immersion, such as those of Nicholson, and Fahrenheit, are but little used outside of laboratories. In actual practice instruments similar to those exhibited in Fig. 17

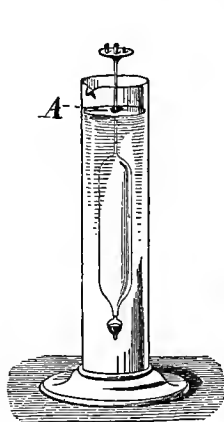


Fig. 16.

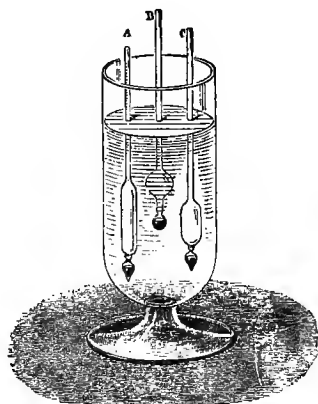


Fig. 17.

are used. They sink to different levels when placed in different liquids, and from the graduations of the scales attached to their stems the specific gravity of the liquid tested may be determined. Instead of indicating specific gravities, these scales are sometimes graduated to indicate percentages of some substance mixed with, or dissolved in a liquid, such as alcohol, sugar, or salt, in water, or butter fat in milk. When so used the instruments are called alcoholometers, densimeters, salimeters, or lactometers, according to the use to which they are to be put.

They are sometimes still further distinguished by means of the name of the manufacturer, those generally used being Beaume's, and Tweedel's. Small, hollow glass balls are sometimes used to find the specific gravity of a liquid. They are made in a graded series, and numbers are cut in the glass, giving their specific gravities, which have been determined by using them in liquids whose specific gravities are known. To find the specific gravity of a liquid, it is only necessary to find which one of the balls will just float in it.

By experimenting with different solutions, and mixtures, the student should make himself thoroughly familiar with the use of these practical instruments.

XI. MOHR'S BALANCE.

Many instruments have been constructed for the rapid determination of the specific gravity of liquids, and of these the Westphal modification of Mohr's balance shewn in Fig. 18 is

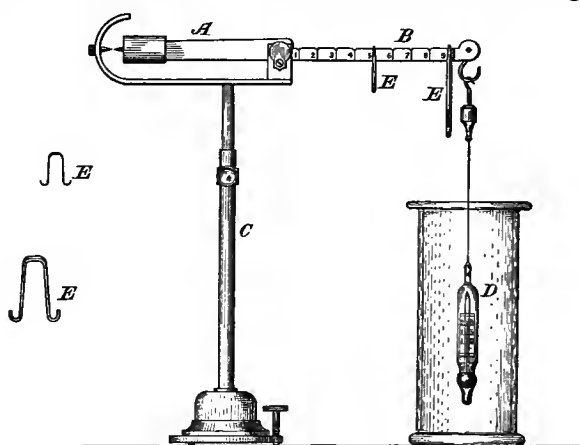


Fig. 18.

probably the most convenient. It consists of a weighing beam *AB* with knife edges resting on the stand *C*, a glass thermometer plummet *D* suspended from it by a platinum wire, and a set

of four riders E, E, E, E , whose weights are 5, .5, .05, and .005 grams respectively. The beam AB is graduated in ten equal divisions, and the glass plummet, whose weight is 15 grams, is of such a volume that it displaces 5 grams of distilled water at 16.7°C .

If, then, equilibrium exists with the plummet attached to the beam in air, it can be readily seen that it will again be established, if the plummet be immersed in distilled water at 16.7°C , by hanging the largest, or unity rider to the hook on the end of the beam. If the plummet be immersed in a liquid lighter than distilled water, the unity rider is removed from the hook, and placed upon that point on the beam which will bring it into equilibrium. If this point lies between two division marks, the unity rider is placed upon the lower of these, and the point of equilibrium is sought by a similar application of the next smaller rider, and also, if necessary, of the two following ones. If it should happen that two riders come to find their places on the same division mark, the smaller weight is hung upon the larger. If when the plummet is immersed in such a liquid the riders in order of size are on the points 9, 2, 7, and 6 respectively, the student can easily deduce from elementary principles of mechanics that the weight of the liquid displaced by the plummet is .9276 that of the distilled water displaced by it; the specific gravity of the liquid examined is then .9276.

In determining the specific gravity of a liquid heavier than water, the unity rider is allowed to remain on the hook with the plummet, while a second unity rider, and the three other smaller riders are used on the beam as described above.

As the instrument illustrates splendidly the principles of mechanics and hydrostatics, the student should make himself thoroughly familiar with its workings.

XII. BOYLE'S LAW.

A. *For pressures greater than one atmosphere.*

In Fig. 19, CD and BF are two glass tubes attached to a stand, and connected at the ends D , and B by a metal tube into which both are cemented. Between them a scale is so placed as to indicate equal volumes in the tube BF . The upper end of this tube is closed by two taps F , and G ; but if the experiment is conducted with a dry gas, only one of these is used, the upper one, G , being then removed.

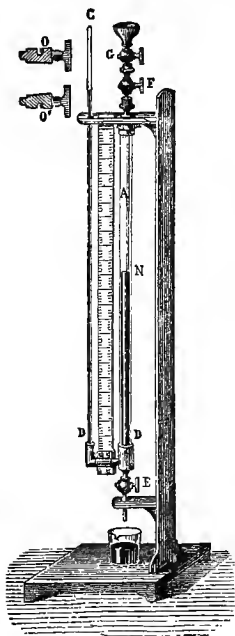


Fig. 19.

Experiment. — Open the tap F so as to allow the air in the tube BF to communicate with that outside, and pour mercury into the open end C of the tube CD until it rises in both tubes to the level of some chosen mark on the scale. By then closing the tap F a mass of air will be inclosed in the tube BF at atmospheric pressure. On pouring more mercury in at C , the volume occupied by the air will be gradually decreased, and the mercury will rise to different heights in the two tubes. Let H be the height of the barometric column, and V the volume occupied by the air at atmospheric pressure. When h is the difference between the heights of the two columns of mercury, let v be the volume of the air. The pressures, therefore, to which

the air is subjected in the two cases are proportional to H , and $H+h$ respectively. If the experiment is conducted carefully, it will be found that $HV=(H+h)v$, which proves the law that the volume of a mass of dry air at constant temperature varies inversely as the pressure to which it is subjected. The tube

BF may be filled with dry air before commencing the experiment, by first filling the two tubes with mercury, and then letting it run out through the tap at the bottom of the metal tube, the air entering BF at the tap F after passing through a drying tube filled with calcium chloride.

The same experiment may be conducted with other gases, and it will be found that they follow the same law. By attaching the tap G (shown in section at O , and O') to the tube BF , we may, on account of its peculiar construction, admit a liquid to the tube drop by drop, and the laws of superheated, and saturated vapours may thus be investigated.

B. For pressures less than one atmosphere.

In Fig. 20, PM , a long, narrow vessel, supported on a stand, is nearly filled with mercury, and AB , a glass tube closed at one end, is graduated to indicate equal volumes.

Experiment.—Partly fill the tube AB with mercury, invert it in the vessel PM , and then lower it until the surfaces of the mercury inside, and outside the tube, are in the same plane. The air in the tube is then at atmospheric pressure. Let V be the volume it then occupies, and H the height of the barometric column. Raise the tube little by little, and note each time the volume the air occupies and the pressure to which it is subjected. In the figure AC is the volume of the air, and CD the height of the mercury in the tube at one stage of the experiment. Since the pressure is the same inside, and outside the tube at the level D , it is readily seen that the pressure to which the air is subjected in this case is proportional to the difference between the height

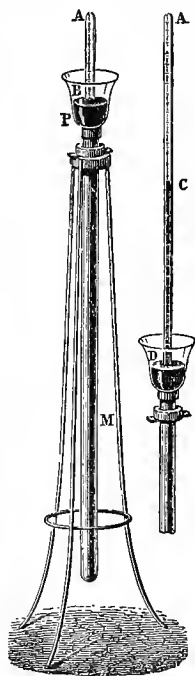


Fig. 20.

CD , and that of the barometer. If CD is denoted by h , this difference is then equal to $H-h$. Here again, denoting the volume AC by v , we will have HV equal to $(H-h)v$, and will thus find that the law holds for pressures less than one atmosphere.

Usually there is also a barometric tube inverted in the mercury in PM , and attached to a support. The quantity $(H-h)$ can then be measured directly, and accurately by means of a cathetometer.

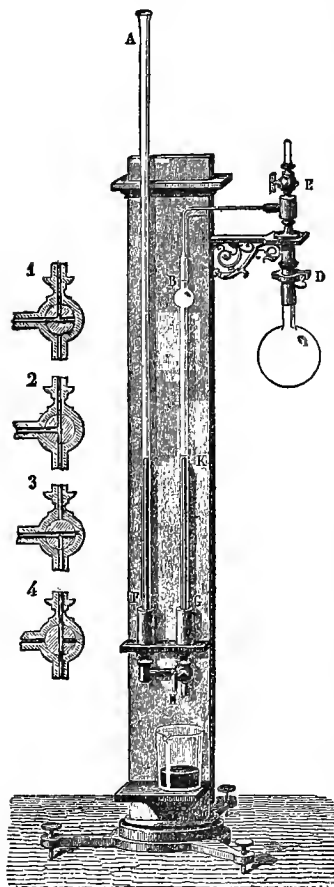


Fig. 21.

XIII. THE VOLUMENOMETER.

This instrument is used to find the volume of a substance, such as gunpowder, sugar, or salt, which cannot be placed in contact with water. In Fig. 21, AF , and BG are two glass tubes containing mercury, whose ends F and G are cemented into a metal tube which connects them. In this tube there is a three-way tap which serves different purposes, according to the position in which it is placed. These are indicated in Figs. (1), (2), (3), and (4).

In (1) communication is established between the two tubes, in (2) the mercury can run out of both tubes, in (3) it can run out of AF only, and in (4) out of BG only.

On the tube BG are two marks, B and K , and between them the tube expands into a little bulb; above B the tube narrows

down, and after being bent is connected to a three-way socket, at one end of which is a tap E , and to the other there is attached a glass globe which can be readily removed, or put on by means of the collar D .

Let v be the volume of the tube between the marks B and K , V that of the tube above B including the globe attached at D , and x the required volume of the given substance. To calculate v , fill the tube BG with mercury up to the mark B , and then let it run out through the tap H until it sinks in the tube to the mark K . By weighing the mercury that runs out v may be determined.

Experiment.—Attach the globe at D , and after opening the tap E fill the two tubes with mercury up to the mark B . Close the tap E , the air thus shut in being at atmospheric pressure, and then allow the mercury to run out of both tubes until it is at the mark K in the tube BG . Let h be the height of the mercury in this tube above that in the tube AF , and applying Boyle's Law, we have therefore, if H is the barometric height,

$$HV = (H - h)(V + v). \quad (1)$$

Now remove the globe, put the given substance whose volume is to be determined into it, and after again attaching it by the collar D , repeat the operation exactly as described above. If h' be now the height of the mercury in one tube above that in the other, and H' that of the barometric column, we have again, by applying Boyle's Law,

$$H'(V - x) = (H' - h')(V - x + v). \quad (2)$$

$$\text{From (1) we have} \quad V = \frac{v(H - h)}{h}, \quad (3)$$

$$\text{and from (2),} \quad V - x = \frac{v(H' - h')}{h'}; \quad (4)$$

∴ from (3) and (4)

$$x = v \left(\frac{H}{h} - \frac{H'}{h'} \right), \text{ or if } H = H',$$

$$x = \frac{vH(h' - h)}{hh'} = vH \left(\frac{1}{h} - \frac{1}{h'} \right). \quad (5)$$

If W is the weight of the substance in grams, and x its volume in cubic centimeters, its specific gravity is given by

$$\text{Sp. Gr.} = \frac{W}{x}$$

The two tubes AF , and BG are usually graduated in centimeters, and in this case h and h' can be read off directly. Otherwise a cathetometer must be used.

XIV. DETERMINATION OF CAPILLARY CONSTANTS.

Various phenomena in nature show that the molecules of a body, whether in a solid or in a liquid state, are subject to the action of two contrary forces, one of which tends to bring them together, and the other to separate them from each other. The

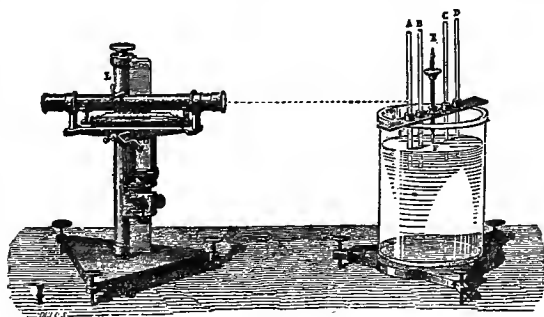


Fig. 22.

former of these is a cohesive or attractive force (quite distinct from gravitation), which acts only at very short distances between particles of the same, or of different matter; while the

latter is due to a vibratory motion which the molecules are supposed to receive on the application of heat. It is with these *attractive* forces, whose presence is the condition for the existence of matter in the solid state, that we wish especially to deal.

That they act only at very short distances is evidenced by the fact, that, if we grind up a solid into a fine powder, the particles, when the matter is in this state, are so far apart that these forces exert no influence; while if we take two pieces of lead or glass with very smooth plane surfaces, and place these in contact, we find that they at once adhere to each other, and it is only on the application of considerable force that we are able to separate them. Although the presence of these forces is chiefly manifested in matter in the solid state, they exert, however, an action between the particles of a liquid, and the general term *capillarity* is used to denote all the phenomena produced by them.

If we dip a clean glass rod into water, we find on withdrawing it that a drop clings to its lower extremity. As gravity tends to detach the particles of the drop from each other, and these again from the glass, we learn that there is an attraction between the particles of water forming the drop, and also one between the particles of glass, and of water, which is greater than that exerted by gravity. Again, if we suspend a glass disc horizontally from one of the arms of a balance, and allow it to come in contact with the surface of water, we find that in order to detach it a considerable number of weights must be placed in the pan attached to the opposite arm. If instead of water we use mercury, exactly the same action takes place, but a smaller weight is necessary to make the separation. If we replace the glass disc by one of copper of exactly the same form, and dimensions, we will find that the same weight as before must be used to detach it from the water, but one different from that used before to make the separation from mercury. The explanation is simple. In the one case a layer of water adheres to the disc, and we separate, not the disc from the water, but a layer of water

from the main body; while with mercury we separate the disc from the liquid. This shews that the attractions between particles of water are less than those between particles of glass, and of water, and it affords an explanation of the term *wetting*. A liquid is said to wet a solid when the particles of the solid attract those of the liquid with a force greater than that exerted between the particles of liquid themselves.

Another class of phenomena caused by these molecular actions is exhibited in the following experiments. If we make a small cup out of a sheet of wire gauze, and, after dipping it in melted paraffine, shake it so that its meshes do not become stopped up, we will find that we can pour into it a large quantity of water before any begins to run out. Or, again, if we rub a common sewing needle with grease so that the water will not wet it, and then place it gently on the surface of clean water, it will float there if not disturbed. These, and many other such experiments, lead us to conclude that at the surface of a liquid there is a thin layer of particles which act in the same manner as a flexible elastic membrane spread over the surface. Although it is difficult to explain just why this outside layer acts in this manner, we can, however, readily understand that the particles near the surface of a liquid are subject to actions different in degree from those on the particles in the interior.

In Fig. 23 let ABC represent a mass of liquid, and P a particle of it situated at the center of a small sphere DE of such a radius that none of the liquid outside of it has any action on the particle at P . If then P is at a point in the liquid whose distance from the surface is greater than the radius of this sphere, it is evident, from symmetry, that the resultant action on the particle is zero. If, however, it is situated at P_1 , it is clear that there is no liquid to compensate the action of the part FGH of the sphere, and there will be, in this case, a resultant action on P_1 depending on the mass FGH , and tending to draw it into the body of the liquid. This will be still more evident if the particle is situated in the surface, at P_2 ; for in

this case there is the action of a hemisphere of liquid not compensated.

We thus see that the particles at, or near, the surface of a liquid are subjected to attractions which tend to draw them into the main body, and the effect on the whole mass is precisely the same as if it were surrounded by a thin elastic membrane.

In the case of liquid films, and soap bubbles we have two sets of surface particles with a thin layer of liquid between them. Delicate experiments have been devised with these to ascertain the force required to separate the particles in the surface layer

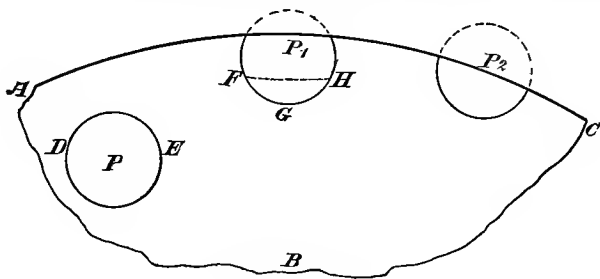


Fig. 23.

from each other by measuring the radii of bubbles just before they break, and by noting the pressures to which they are subjected; but probably it can best be found in the case of liquids which *wet* glass by the use of capillary tubes. Figure 22 shows a glass vessel nearly filled with such a liquid in which are suspended several capillary tubes *A*, *B*, *C*, and *D*, a sharp-pointed screw *E* which can be raised or lowered, and a cathetometer *L*. If the liquid be drawn up through one of these tubes, and then allowed to fall back, it will leave a layer of its particles adhering to the glass on the inside, and will not sink to the level of the outside liquid, but will remain stationary at a height depending on the radius of the tube.

Since the hydrostatic pressure inside the tube is the same as that outside for points in the same horizontal plane, then the pressure inside just below the surface layer of particles is less

than that at the surface of the main body of the liquid, and we may therefore look upon the column as being held up by the attractions between the particles along the periphery of the surface layer, and those adhering to the tube. If, then, T denotes the value of this surface tension per unit length, h the height of the column, and r its radius, we have, since the angle at which the surface of the liquid meets the tube is zero,

$$2 \pi r T = \pi g \rho r^2 h,$$

or

$$T = g \frac{\rho}{2} \cdot r h.$$

As this tension T is exerted between particles of the same kind of matter, it should have a constant value whatever may be the radius of the tube. The product, $r h$, therefore should be a constant, and this may be illustrated by using tubes of different sizes.

In order to ascertain the height h at which the liquid stands, lower the screw E until it just grazes the liquid, and then with the cathetometer find the vertical distance between the surface of the liquid in the tube, and the upper extremity of the screw. The difference between this length, and that of the screw, which has been previously determined, will give h .

The radius of the tube r is found by running a thread of mercury into it. If l be the length of this thread in centimeters, w its weight in grams, and d the density of mercury, then

$$\pi r^2 l d = w,$$

or

$$r = \sqrt{\frac{w}{\pi l d}}$$

This method of finding the surface tension may be adopted in the case of such liquids as water, alcohol, or sulphuric acid, and in order to obtain uniform results, the greatest care must be taken to have tubes, and liquid perfectly clean.

There are many instructive experiments in capillarity, and students should especially make themselves familiar with those

devised by Plateau on liquid films, and on the forms that a mass of liquid will take when freed from the action of gravity.

A full account of the various elementary phenomena of capillarity is to be found in a little book entitled "Soap Bubbles" by C. V. Boys.

XV. THE SEXTANT.

This instrument, shewn in Fig. 24, consists of a framework in the shape of a circular sector, of which the arc GH is graduated. About the center of this sector turns an index arm EF provided with a vernier and tangent screw. At the center a mirror M is fixed normally to this arm, and moves with

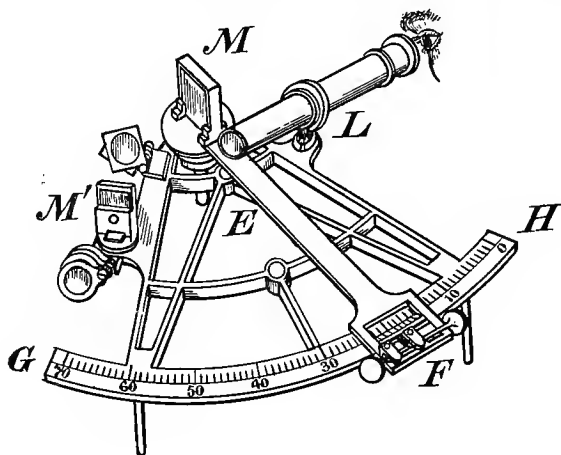


Fig. 24.

it; while to the frame on the left-hand side is attached a second mirror M' , which has only the lower half silvered, and may by means of screws be rotated about an axis either parallel, or perpendicular to the frame. Opposite to this mirror, in most sextants, there is attached permanently to the frame a telescope, while in others a small hole in a screen at L is substituted for it.

Experiment. — The sextant is used to measure angles subtended at the eye by two distant objects. Before taking a measurement it is necessary to know that the two mirrors are parallel, and this is arrived at by first fixing the sliding arm at

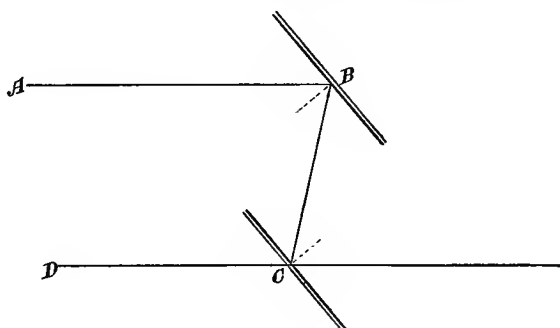


Fig. 25.

zero, and then looking through the telescope and the unsilvered glass M' at some very distant object.

If the two mirrors are parallel, the observer will notice that immediately below the portion of the object viewed directly he

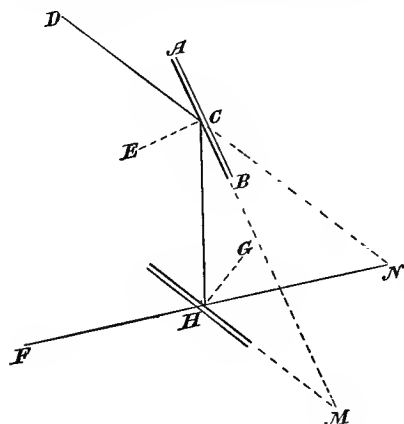


Fig. 26.

will see an image of the other part by means of rays of light coming from it after having been twice reflected at the mirrors M and M' . This will be evident from Fig. 25, in which AB and DC are rays coming from a very distant object. If this is not the case, then the mirrors are not parallel, but are made so by turning the mirror M' by the screws mentioned previously.

This done, hold the sextant so that its plane may pass through both the objects whose angular distance is to be

measured; then look through the telescope and the unsilvered glass at the object to the left, and turn the arm EF until the eye sees the image of the second object, after the rays forming it have been twice reflected, coincide with the object seen directly. The angular distance between the objects is then twice the angle between the mirrors, and therefore twice the angle through which the movable arm has been turned, as is evident from the following proof.

In Fig. 26 D and F are two distant objects, BA and H are the two mirrors M and M' , and DC and FH are rays of light coming from the objects D and F respectively.

The angle DNF = the angle DCH – the angle NHC
 $= 2$ (angle ECH – angle GHC)
 $= 2$ (angle $ECH + 90^\circ$ – angle $GHC - 90^\circ$)
 $= 2$ (angle ACH – angle CHM)
 $=$ twice angle HMC , which is the angle between the mirrors.

Besides measuring the altitude of the sun, the angular distance between two stars, or that between two very distant objects, a very instructive exercise with the sextant is to measure the angle subtended at the eye by the sun's or the moon's disc in various positions in the heavens. In measuring the sun's diameter, dark glasses may be placed in front of the mirrors M and M' . The sextant may also be used to find the height of a tower whose base is supposed inaccessible.

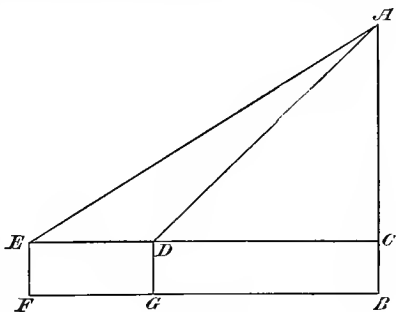


Fig. 27.

In Fig. 27 let AB represent the tower, and C a point on it in the same horizontal line as the observer's eye. Then since

$$\cot AEC = \frac{EC}{AC}, \text{ and } \cot ADC = \frac{DC}{AC},$$

$$\cot AEC - \cot ADC = \frac{EC - DC}{AC},$$

$$\therefore AC = \frac{ED}{\cot AEC - \cot ADC}.$$

So that if the observer, while in the position EF , measures the angle AEC , subtended at the eye by AC , and then moves up a known distance ED , and observes the angle ADC , he can, by applying the above formula, find the length AC , and adding to this the height of the observer, that of the tower may be obtained. The above formula may be easily adapted to logarithmic calculation.

XVI. THE GONIOMETER.

Figure 28 shews this instrument, which is adapted to measure the angle of a crystal. It consists of a divided circle, which has at its center a small glass platform concentric with it, and capable of an independent rotation.

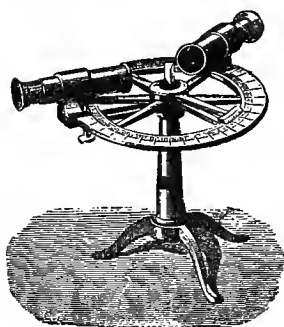


Fig. 28.

The instrument is provided with a collimator, at one end of which is an adjustable slit which can be illuminated, and at the other there is inserted a convex lens to cause the rays coming from the slit to issue parallel to each other. There is also a telescope, attached to a sliding arm, that can be

rotated about the axis of the divided circle. To the arms of the small glass platform, and the telescope are attached verniers which generally permit readings to be taken to minutes.

METHOD I.

Theory. — In Fig. 29 AD is a beam of parallel light, which on striking the angle EDF of the crystal at right angles to the edge is broken up into two beams which are reflected along DB and DG .

By the laws of reflection, the beam BD has been deviated from its original path AD through an angle equal to twice the angle between AD and ED produced, and also the beam DG through twice the angle between AD and FD produced. Therefore the angle between BD and GD is equal to twice the sum of the angles made by ED and FD produced, with AD , which is twice the angle of the crystal.

Experiment. — Place the crystal upon the central glass platform so that its edge is in the axis of the graduated circle, and so turned that the light from the illuminated slit, after passing through the collimator, strikes upon this edge, and being split up into two beams is reflected from each of the two faces forming the angle. This may be accomplished by trial with the telescope. Turn the telescope so as to intercept one of these rays, and note the reading on the divided circle; then turn it until it receives the other reflected beam, and again note the reading. The difference between these two readings is twice the angle of the crystal. From the theory the student will readily see that great care must be taken, if reliable work is desired, to have the edge of the crystal placed accurately over the axis of the circle, and perpendicular to it. This condition of perpendicularity may be obtained by noting when the edge of the crystal is in line with its image seen reflected in the glass platform.

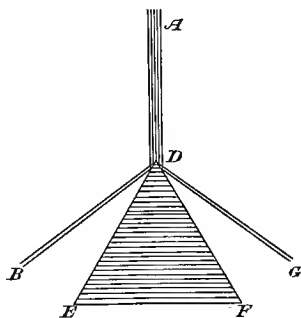


Fig. 29.

It is also desirable to choose a part of the edge for the light to fall on which has the smallest number of flaws in it, or else it will be difficult to take the telescope readings with accuracy.

METHOD II.

Theory. — In Fig. 30 AB is a beam of parallel light incident upon the edge of the crystal DBE at the angle B , BC is the path taken by this beam when reflected by the face BE , and $D'BE'$ is the same crystal in its second position, with its face $D'B$ now in the plane formerly occupied by BE' . From this it is evident that the ray on striking the face $D'B$ will be again reflected along BC .

Now since BE is in the same straight line as $D'B$, it is clear that, as DB in the first position corresponds with $D'B$ in the second, the crystal has been turned through an angle equal to 180° , less the angle DBE , so that if we measure the angle through which the crystal is turned, and subtract this from two right angles, we get the angle of the crystal.

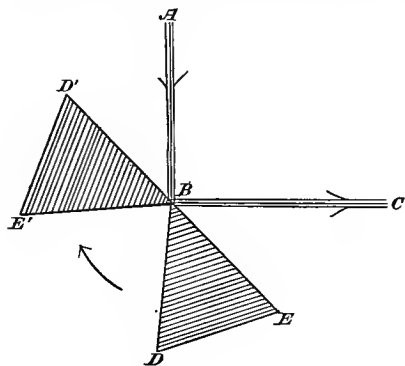


Fig. 30.

Experiment. — Place the crystal with its edge as indicated in the previous experiment; then turn the telescope until it receives the ray BC , and clamp it there. Then note the reading on the divided circle as indicated by the vernier on the arm attached to the platform on which the crystal rests. After this turn this arm until, on looking through the telescope, the observer again sees the light reflected along BC . This indicates that the crystal is in the second position $D'BE'$, and by again noting the reading on the divided circle the angle through

which the crystal has been turned may be found. To secure accurate results the same precautions must be taken as in finding the angle by the previous method.

XVII. RADIUS OF CURVATURE OF A CONCAVE SPHERICAL MIRROR.

Theory. — In Fig. 31 DHE is a section of a concave spherical mirror whose center is C . P is the illuminated point, P'

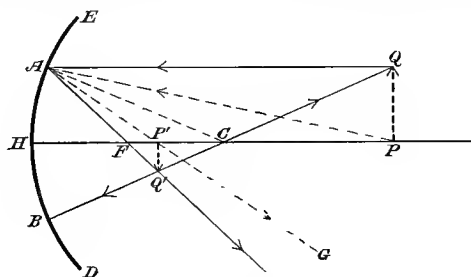


Fig. 31.

its image, and PA , AG are the incident and reflected rays. By the law of reflection, the angle PAC is equal to the angle CAG , and

$$\therefore \frac{\sin PAC}{\sin ACP} = \frac{\sin P'AC}{\sin ACP'} \quad \therefore \frac{PC}{AP} = \frac{P'C}{AP'}$$

Now limit the ray PA by supposing it incident close to H , the point where PC produced meets the mirror. Then AP and AP' will become equal to HP and HP' respectively,

$$\begin{aligned} \therefore \frac{PC}{PH} &= \frac{P'C}{P'H}, \\ \therefore \frac{PH - HC}{PH} &= \frac{HC - P'H}{P'H}, \\ \therefore \frac{1}{PH} + \frac{1}{P'H} &= \frac{2}{HC}; \end{aligned}$$

or, denoting PH , $P'H$, and HC by p , p' , and r respectively, we obtain the formula $\frac{1}{p} + \frac{1}{p'} = \frac{2}{r}$, connecting the distances of the image and the object from the mirror with the radius of curvature.

Experiment. — Place an illuminated object before the mirror, and receive its image on a screen situated so as not to cut off all the rays issuing from P . Then measure p and p' directly, and substituting their values in the formula $\frac{1}{p} + \frac{1}{p'} = \frac{2}{r}$, obtain the value of r . The experiment may be varied by giving p values ranging from infinity to $\frac{r}{2}$, noting particularly the case where $p=r$, in which case p' also equals r , and image and object coincide. For values of p less than $\frac{r}{2}$, it is evident from the formula that p' is negative. The physical signification of this is that the image is then virtual, and so can no longer be received on a screen.

For an illuminated object, a candle or small gas flame may be used, or, better still, a fine platinum wire suspended in a Bunsen flame, or else a dark thread stretched across a small illuminated opening in a screen.

For ordinary purposes it will be sufficient to make the measurements with a millimeter scale, but, for greater accuracy, place the illuminated object and the mirror on stands which slide freely on a bench with scale attached. By using verniers on these stands very close results can be obtained. A very useful exercise in connection with this experiment is the determination of the radii of curvature of small mirrors, such as are used in reflecting telescopes, or galvanometers.

XVIII. RADIUS OF CURVATURE OF A CONVEX SPHERICAL MIRROR.

Theory. — In Fig. 32 ABA' is a section of a convex spherical mirror whose center is C . P is the position of a lighted candle,

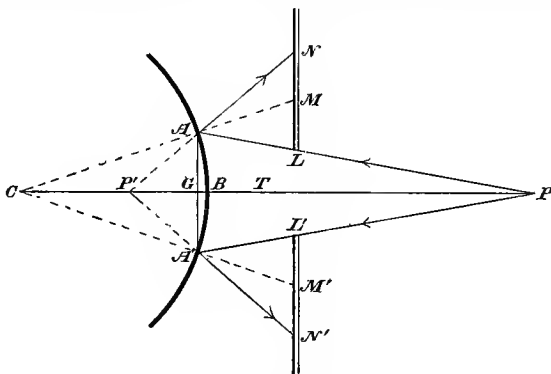


Fig. 32.

or gas jet, P' its image, and PA , AN are the incident and reflected rays.

By the law of reflection, the angle MAL is equal to the angle MAN , and therefore also equal to the angle $P'AC$.

$$\therefore \frac{\sin PAC}{\sin ACP} = \frac{\sin P'AC}{\sin ACP}, \quad \therefore \frac{PC}{PA} = \frac{P'C}{P'A}.$$

Now limit the ray PA by supposing it incident close to B , the point where PC cuts the mirror. Then PA and $P'A$ will become PB and $P'B$ respectively, and

$$\begin{aligned} \frac{PC}{PB} &= \frac{P'C}{P'B}, \\ \frac{PB+BC}{PB} &= \frac{BC-P'B}{P'B}, \\ \therefore \frac{1}{P'B} - \frac{1}{PB} &= \frac{2}{BC}; \end{aligned}$$

or, denoting PB , $P'B$, and BC by p , p' , and r respectively, we have for a formula connecting the distances of the object and the image from the mirror, with the length of the radius,

$$\frac{1}{p'} - \frac{1}{p} = \frac{2}{r}$$

Experiment. — Place the mirror in an upright position. Then cover the face of it with a sheet of paper having two holes in it, denoted by A and A' , at equal distances from the center of the surface of the mirror. It will probably also be found most convenient to have them in the same horizontal plane as the center. By this arrangement there will be two rays of light, AN and $A'N'$, reflected from the mirror. Intercept these rays by a screen NN' so made as to permit this, and yet not cut off all the light coming to the mirror. The measurements then to be taken are PB , the distance between the candle and the mirror, BT that of the screen from the mirror, AA' that between the holes in the paper, and NN' that between the spots of light formed by the intercepted rays on the screen.

Then, since the triangles $P'AG$ and $P'NT$ are similar, we have $\frac{P'G}{GA} = \frac{P'B+BT}{TN}$, or if AA' is very small,

$$\frac{P'B}{AB} = \frac{P'B+BT}{TN},$$

$$\therefore P'B = \frac{AB \cdot BT}{TN - AB} = \frac{AA' \cdot BT}{NN' - AA'}$$

Therefore $P'B$ or p' can be found, and having measured BP , or p directly, we can, by substituting the values of these two quantities in the formula $\frac{1}{p'} - \frac{1}{p} = \frac{2}{r}$, obtain r .

In order to obtain accurate results in this experiment, the two holes in the paper must be taken as close together as circumstances will permit; and, as it is somewhat difficult to measure accurately the distance between the spots of light, it will be

well to have the holes in the paper very small, and to take great care to have their edges neatly cut.

Note on the construction of images. — By an extension of the theory given in the two preceding experiments, we may find the position of the image of an object which is not a point, as we there supposed it to be. In Fig. 31 let PQ be the object. Join QC , and produce it to meet the section DHE in B . Then just as we found P' , the image of P , to lie on PH , and to be

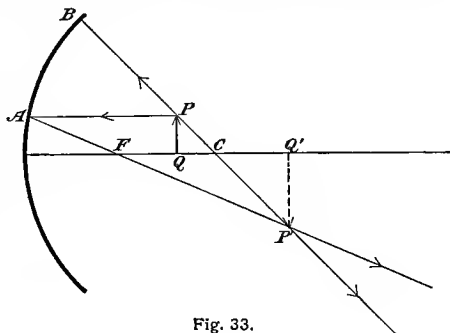


Fig. 33.

given by the formula $\frac{1}{PH} + \frac{1}{P'H} = \frac{2}{HC}$, so Q' , the image of Q , must lie on QB and be given by the relation $\frac{1}{QB} + \frac{1}{Q'B} = \frac{2}{BC}$.

By joining Q' and P' as thus found, $Q'P'$ will be the image of QP . From the above explanation it will be readily seen that $Q'P'$ will not be quite perpendicular to either PH , or QB , and it is therefore for this reason that objects as seen in a spherical mirror appear distorted. From the formula $\frac{1}{p} + \frac{1}{p'} = \frac{2}{r}$, it will

be seen that if the object be placed off at an infinite distance the image will be at a distance $\frac{r}{2}$ from the mirror. This point is called the *focus*, and as all rays coming to the mirror from a point at an infinite distance are practically parallel, we may define it to be the point through which pass all the reflected rays which are incident on the mirror parallel to the axis, PH . Thus we see the ray QA , which is parallel to PH , will, when reflected, approximately pass through the focus F which bisects CH . Again, a ray QC which passes through the center will, when reflected, come back on itself. The point where this

ray intersects the ray AF will be the image of Q , and by dropping a perpendicular from this point on the axis PH , we will

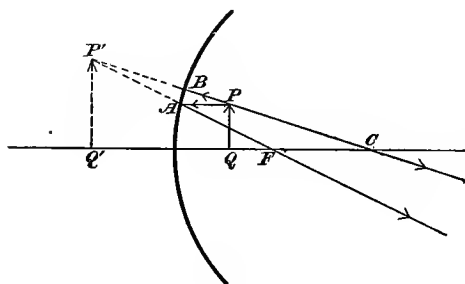


Fig. 34.

approximately have the position of $P'Q'$ the image of PQ .

This method of constructing images is exhibited in Figs. 31, 33, and 34, and it will be an instructive exercise for the student to apply the same method to the case of a convex mirror.

XIX. RADIUS OF CURVATURE OF SPHERICAL MIRRORS.

GENERAL METHOD.

Theory. — In Fig. 35 HDG is a section of a convex mirror whose center is C ; A and B are two illuminated objects; A' and B' are their images; EF is a finely divided scale placed before

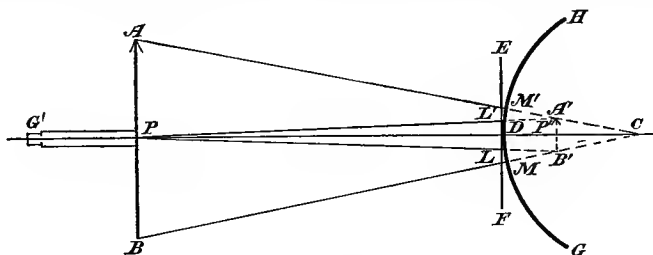


Fig. 35.

the mirror in the same horizontal plane as A and B ; and PG' , a telescope placed in the line joining C to the point midway between the objects A and B , has its objective in the line AB . L and L' are the points where the lines $A'P$ and $B'P$ cut EF .

The formula connecting image and object for convex mirrors is $\frac{1}{p} = \frac{1}{p'} - \frac{2}{r}$; and since A' , B' , and P' are the images of A , B , and P respectively, we have denoting LL' by y , AB by x , $A'B'$ by x' , PD by p , $P'D$ by p' , and PP' by y' ,

$$\frac{1}{p} = \frac{1}{p'} - \frac{2}{r}; \quad (1)$$

and by comparing the sizes of the image and of the object, we have

$$\frac{x'}{x} = \frac{CP'}{CP}. \quad (2)$$

From equation (1) we have

$$\begin{aligned} \frac{1}{p} + \frac{1}{r} &= \frac{1}{p'} - \frac{1}{r}, \\ \frac{r+p}{p} &= \frac{r-p'}{p'}, \end{aligned} \quad (3)$$

$$\therefore \frac{p'}{p} = \frac{r-p'}{r+p} = \frac{CP'}{CP} = \frac{x'}{x}, \quad \therefore x' = \frac{xp'}{p}$$

Again, from (3),
$$p' = \frac{rp}{r+2p}, \quad (4)$$

$$\therefore x' = \frac{xr}{r+2p}. \quad (5)$$

From the figure
$$\frac{x'}{y} = \frac{y'}{p}, \quad \therefore y = \frac{px'}{y'},$$

$$\therefore y = \frac{p}{y'} \cdot \frac{xr}{r+2p}, \quad \text{from (5)}$$

$$\therefore y = \frac{xr p}{\left(r+2p\right)\left(p+\frac{rp}{r+2p}\right)}, \quad \text{from (4)}$$

$$= \frac{rx}{2(r+p)},$$

$$\therefore r = \frac{2py}{x-2y}. \quad (6)$$

By a process of reasoning similar to this, the student may easily find for himself that in the case of concave mirrors the formula becomes

$$r = \frac{2py}{x+2y}. \quad (7)$$

Experiment. — If this experiment is performed in the dark, two lighted candles, or gas jets may be used for the illuminated points A and B ; and in case the scale FE is not sufficiently illuminated to be easily read, it may be made so by projecting a beam of light on it by means of a convex lens. However, it is better to perform the experiment in a well-lighted room, and to use two porcelain, or ivory scales with divisions made in black, the one to take the position FE , and two marks on the other to replace the illuminated points A and B . Having then placed the mirror, telescope, illuminated objects, and scales in the positions indicated, adjust the telescope so as to be able to read the scale FE . If PD is taken of considerable length, A' and B' will be approximately *in focus* at the same time as FE , and the length LL' on the scale, cut off by the space between A' and B' , may thus be directly read off by means of the telescope. The lengths p and x are measured with ordinary scales, and for accuracy it would be well to check the work by giving these quantities different values.

This method may also be used to find the radius of curvature of the surface of a lens. In this case each face of the lens will give a pair of images. By noting that the images produced by a convex reflecting surface are always erect, the student will find no difficulty in distinguishing the two sets.

XX. FOCAL LENGTH OF A BICONVEX LENS.

METHOD I.

Theory. — In Fig. 36, AC is a biconvex lens whose focus is at F , PP' is the principal axis, PQ is an illuminated object, and $P'Q'$ its image. If we assume that a ray of light striking the lens at its center passes through without deviation, and that

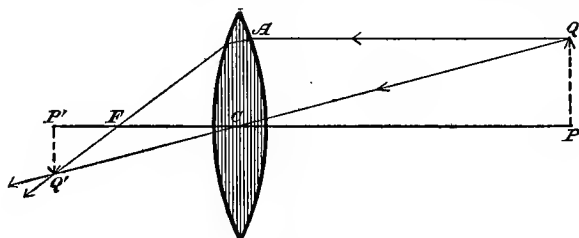


Fig. 36.

a ray parallel to the principal axis on striking it ultimately passes through the focus, we are able to obtain a relation connecting the distance of the object, and of its image from the center of the lens C , with the focal length.

If QC and QA are these two rays, the point Q' where they intersect will be the image of Q , and dropping a perpendicular $Q'P'$ on PP' , $P'Q'$ will be the image of PQ .

Denoting CP by p , CP' by p' , and CF by f , we have, from similar triangles,

$$\frac{PQ}{P'Q'} = \frac{p}{p'} \quad (1)$$

Again, by erecting at C a line perpendicular to PP' , and terminated at the point where it cuts the ray QA , we have

$$\frac{PQ}{P'Q'} = \frac{f}{p' - f}; \quad (2)$$

$\therefore$ from (1) and (2)

$$\frac{p' - f}{f} = \frac{p'}{p}, \quad \therefore \frac{1}{p'} + \frac{1}{p} = \frac{1}{f}. \quad (3)$$

Experiment. — Having placed the object and the lens in the same horizontal line, move the screen until the image is distinctly focussed on it. Then measure directly the distance of the image and of the object from the center of the lens, *i.e.* p and p' , and substituting the values of these quantities in the formula $\frac{1}{p'} + \frac{1}{p} = \frac{1}{f}$, obtain f , the focal length. As in the case of concave mirrors, although a lighted candle, or gas jet may

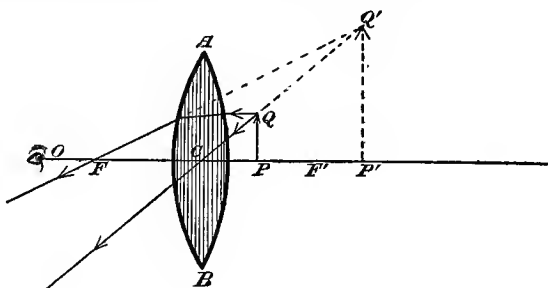


Fig. 37.

be taken for the illuminated object, it will be found that more accurate results are obtained when an incandescent platinum wire, or a dark thread stretched over an illuminated opening in a screen is used. If this experiment is conducted in a dark room, it will be found that a screen painted white or one made of ground glass will be the most suitable. Great care should be taken to have the image well defined.

If the object be at a great distance, the formula indicates that the image will then be at the focus, and the focal length may then be measured directly. If p be taken equal to p' , each is then equal to $2f$, and in that case, the distance between object and screen need only be measured. If p be taken less than f , the formula indicates that p' becomes negative, which signifies that the image is then virtual. This is the case when a convex lens is used as a simple microscope, and Fig. 37 then indicates the relative positions of the image and the object.

METHOD II.

In Fig. 38 P is an illuminated object, L a biconvex lens, and P' the position of a screen with the image of P clearly defined on it. From the symmetry of this arrangement, as well as from



Fig. 38.

an inspection of the formula $\frac{1}{p'} + \frac{1}{p} = \frac{1}{f}$, it can be readily seen that if the lens be moved from L to L' so that $P'L'$ is equal to PL , there will again be an image of P on the screen at P' . Denoting then the lengths PP' and LL' by l and a respectively, we have

$$p' + p = l,$$

$$p' - p = a;$$

or

$$p' = \frac{l+a}{2} \text{ and } p = \frac{l-a}{2}.$$

Substituting these values of p and p' in the formula

$$\frac{1}{p'} + \frac{1}{p} = \frac{1}{f},$$

we have

$$f = \frac{l^2 - a^2}{4l}. \quad (1)$$

Experiment. — Place the illuminated object and the lens in position at some chosen distance apart, which, as is evident from Fig. 37, must be greater than the focal length of the lens. Then adjust the screen so that the image of P is quite distinct on it, and measure PP' , or l the distance between the object and the image. Measure also the distance a through which the lens must be moved in order to have an image of P focussed on the screen a second time, and substituting these values of l and a , in formula (1), find f the focal length of the lens.

This method is especially applicable in finding the focal lengths of combinations of lenses such as the objectives and eyepieces of optical instruments, or those used in photographic cameras.

NOTE. — If non-achromatized lenses are used in these experiments, it is generally difficult for a beginner to decide where the screen should be really placed.

In Fig. 39 AB represents a simple non-achromatized lens, P an illuminated point, and PA and PB the limiting rays of white light which strike the lens. These, when refracted

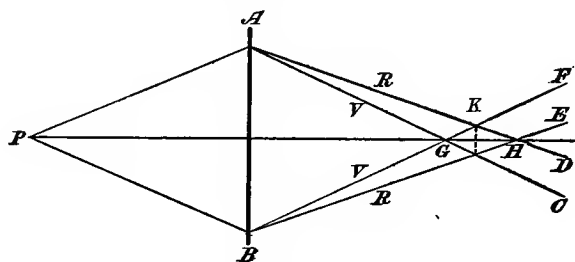


Fig. 39.

through the lens, become dispersed, and two pencils of coloured rays, DAC and FBE , are obtained. AD and BE will therefore be red rays, and AC and BF violet, since the latter are more refracted than the former. It will be readily seen from the figure that any point between G and H may be said to be the image of P . If the screen be placed at G , the image on it will be fringed with a reddish tint, while if placed at H it will have a violet border. Between these two points then there will be a point K where the red and the violet rays overlap, and the image will be of one colour. The screen should therefore be placed at this point.

XXI. FOCAL LENGTH OF A BICONCAVE LENS.

METHOD I.

Theory. — In Fig. 40 AC is a biconcave lens whose focus is at F , PP' is the principal axis, PQ is an illuminated object, and $P'Q'$ is its image. If we assume, as in the case of a convex lens, that a ray, on striking the lens at the center, passes through without deviation, and that one parallel to the principal axis appears ultimately to diverge from the focus, we may also in

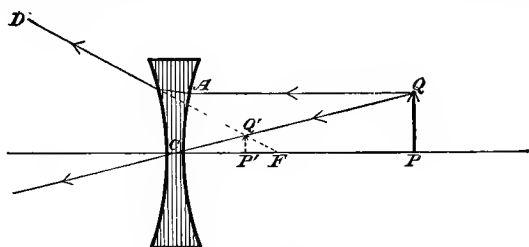


Fig. 40

this case obtain a formula connecting the distance of the image and of the object from the center of the lens C , with its focal length. QA and QC are two such rays, and as on emerging they appear to come from Q' , therefore $Q'P'$, the perpendicular from Q' on PC , will be approximately the image of QP . Denoting PC by p , $P'C$ by p' , and FC by f , we have from similar triangles,

$$\frac{PQ}{P'Q'} = \frac{p}{p'} \quad (1)$$

If a line perpendicular to PP' be erected at C to meet the ray QD , we will have

$$\frac{PQ}{P'Q'} = \frac{f}{f - p'} \quad (2)$$

From (1) and (2)
$$\frac{f - p'}{f} = \frac{p'}{p}, \quad (3)$$

i.e.
$$\frac{1}{p'} - \frac{1}{p} = \frac{1}{f}, \quad (4)$$

which is the formula required.

It will be seen, in this case, that the image is always virtual, and so cannot be measured directly. The following method of finding the focal length is generally adopted.

In Fig. 41 AB is the lens, one of whose surfaces is covered with paper pierced by two small holes, or slits at E and F , which

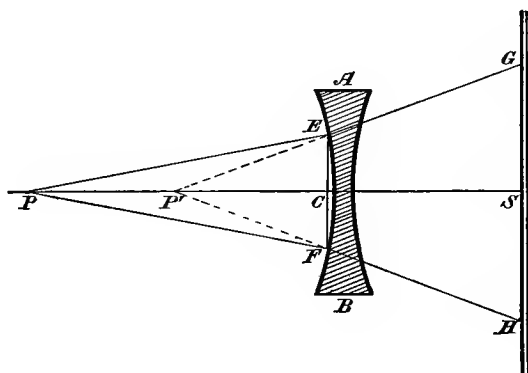


Fig. 41.

permit two pencils of light PE and PF to pass through the lens, and to form two bright spots, G and H , on a screen placed behind it. As these pencils on issuing from the lens appear to come from the point P' , it is therefore the image of P .

From similar triangles, GHP' and $EF P'$, we have approximately,

$$\frac{GH}{P'C + CS} = \frac{EF}{P'C},$$

or

$$P'C = \frac{EF \cdot CS}{GH - EF} \quad (1)$$

If we therefore measure the distance between the slits E and F , and that between the spots of light on the screen G and H , and also measure CS , the distance between the screen and the lens, we may, by substituting in equation (1), find $P'C$, *i.e.* p' . Having found PC , or p directly, we may then, by means of the

formula $\frac{1}{p'} - \frac{1}{p} = \frac{1}{f}$, find f the focal length of the lens.

Experiment.—The results in this experiment may be checked by varying the distance between the holes, or slits in the paper, that between the screen and the lens, or that between the illuminated point and the lens. A lighted candle, or gas jet, will be sufficient for an illuminated object, and best results are obtained when the slits *E* and *F* are made very small and taken close together.

METHOD II.

Theory.—The focal length of a biconcave lens may also be found by using it in combination with a biconvex sufficiently powerful to make the two together act as a convex lens. In

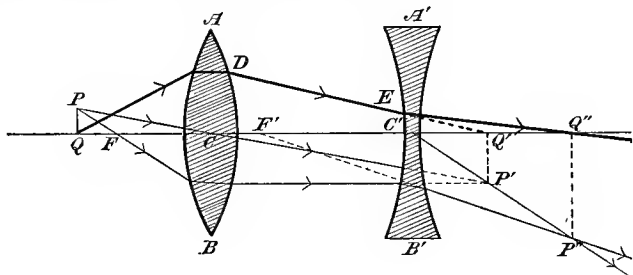


Fig. 42.

Fig. 42 AB is a biconvex lens with center C and focal length FC , $A'B'$ is a biconcave lens with center C' and focal length $F'C'$, PQ is an illuminated object, $P'Q'$ the image that would be formed, were the concave lens removed, and $P''Q''$ is the image of PQ that is formed by the combination; while the heavy line joining Q and Q'' is the actual path of a ray through both lenses. From the figure it will be evident that Q'' is the image of Q' with respect to the lens $A'B'$, because the ray DE , which takes the direction EQ'' with the lens $A'B'$ interposed, would follow the path EQ' , were it removed.

By the principle of reversion applicable in the study of optics, if Q'' were an illuminated point, and $Q''E$ were a ray of light

coming from it, then this ray, after passing through the lens $A'B'$, would take the path ED , and Q would be the image of Q'' .

Since $\frac{1}{p'} - \frac{1}{p} = \frac{1}{f}$ is the formula for a concave lens, the following equation will hold :

$$\frac{1}{C'Q'} - \frac{1}{C'Q''} = \frac{1}{f}$$

If then $C'Q'$ and $C'Q''$ can be found, f , the focal length, may be determined.

Experiment. — Take for an illuminated object an incandescent platinum wire ; then place the convex lens at a certain distance from it, which is to remain the same throughout the experiment. On a screen placed at Q' receive $P'Q'$, the image of PQ formed by the single lens. Measure CQ' , and after removing the screen interpose in the path of the rays the concave lens $A'B'$, and again receive on the screen $P''Q''$, the image of PQ formed by the combination. Measure $C'Q''$ and the distance between the centers of the lenses CC' . Then since $C'Q' = CQ' - CC'$, we can obtain $C'Q'$, and having measured $C'Q''$ directly, we may, by substituting these values in the equation $\frac{1}{C'Q'} - \frac{1}{C'Q''} = \frac{1}{f}$, obtain f , the focal length of the lens $A'B'$.

The results obtained may be checked by varying the distance between the lenses, or that of the incandescent wire from the convex lens. In this experiment the student may meet with some difficulty in deciding where the screen should be placed, owing to the lenses not being achromatic ; but by noting the precautions given in Experiment XX. this may be easily overcome.

XXII. INDICES OF REFRACTION.

Theory. — When a small pencil of white light strikes a prism, as SE in Fig. 43, it is, owing to the unequal refrangibility of the rays composing it, broken up into a series of them, each

corresponding to a different colour. In this experiment, as we wish to deal only with a single ray, suppose SE to be a simple ray obtained from a source of monochromatic light.

When the ray SE strikes the prism at E , it is refracted, and passing through the prism along EF is again refracted at F , and emerging takes the direction FR .

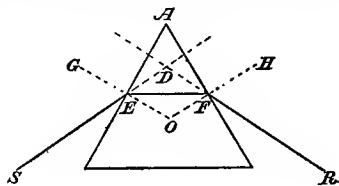


Fig. 43.

EO and FO are normals to the faces of the prism at E and F respectively. Designating the angles SEG and RFH by i and i' , and the angles FEO and EFO by r and r' , we have, from the law of refraction,

$$\sin i = \mu \sin r, \quad (1)$$

$$\sin i' = \mu \sin r'. \quad (2)$$

Also, since the angle EOF is the supplement of the angles r and r' , we have

$$r + r' = A. \quad (3)$$

Again, denoting the angle of deviation by D , we have

$$D = DEF + DFE$$

$$= i - r + i' - r'$$

$$\text{from (3)} \quad = i + i' - A. \quad (4)$$

From (1), we have

$$\sin i = \mu \sin r;$$

$\therefore$ from (3) we have

$$\sin i = \mu \sin (A - r') = \mu \sin A \cos r' - \mu \cos A \sin r'$$

$$= \mu \sin A \cos r' - \cos A \sin i',$$

$$\therefore \{\sin i + \cos A \sin i'\}^2 = \mu^2 \sin^2 A \cos^2 r',$$

$$\begin{aligned} \therefore \{\sin i + \cos A \sin i'\}^2 + \sin^2 A \sin^2 i' \\ = \mu^2 \sin^2 A \cos^2 r' + \mu^2 \sin^2 A \sin^2 r' \end{aligned}$$

from (2);

$$\therefore \sin^2 i + 2 \cos A \sin i \sin i' + \sin^2 i' = \mu^2 \sin^2 A.$$

$$\text{Now } \sin^2 i + \sin^2 i' = \frac{1 - \cos 2i}{2} + \frac{1 - \cos 2i'}{2}$$

$$= 1 - \frac{1}{2} \{ \cos 2i + \cos 2i' \}$$

$$= 1 - \cos(i-i') \cos(i+i'),$$

$$\text{and since } \cos(i-i') - \cos(i+i') = 2 \sin i \sin i',$$

$$\therefore \mu^2 \sin^2 A = 1 - \cos(i-i') \cos(i+i')$$

$$+ \cos A [\cos(i-i') - \cos(i+i')];$$

$$\therefore \text{ since } \sin^2 A + \cos^2 A = 1,$$

$$(\mu^2 - 1) \sin^2 A = \{ \cos A + \cos(i-i') \} \{ \cos A - \cos(i+i') \};$$

i.e. the product of the two quantities on the right-hand side is a constant quantity. Now from (4) we see that D is least when $(i+i')$ is least, *i.e.* when $\cos(i+i')$ is greatest, *i.e.* when $\cos A - \cos(i+i')$ is least; and since this quantity multiplied by $\cos A + \cos(i-i')$ equals a constant quantity, therefore when D is least, $\cos A + \cos(i-i')$ is greatest, *i.e.* D is least when $\cos(i-i')$ is greatest, *i.e.* when $i-i'=0$. When D , therefore, is a minimum, i is equal to i' . The deviation is a minimum when the ray SE strikes the prism so that the angle of incidence equals the angle of emergence. Now when $i=i'$, we have $r=r'$, and $D=2i-A$, and $A=2r$;

$$\therefore \text{ since } \sin i = \mu \sin r,$$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin \frac{D+A}{2}}{\sin \frac{A}{2}}.$$

If, therefore, we are able to get the angle of minimum deviation D , and have found the angle of the prism A , the index of refraction μ may readily be obtained from this formula.

Experiment.—Place the prism on the glass platform attached to the divided circle described in Experiment XVI. in such a way as to allow the light from the illuminated slit to fall on one of its faces EA near the edge A , and move the telescope until it intercepts the emerging ray. Then it will be noticed, if the prism be slightly turned in one direction, that the angle of deviation is increased, while if it is turned in the opposite direction, this angle will be diminished. Continue then to turn it in this latter direction and following the displaced ray with the telescope, it will be noticed that at a certain instant the emerging ray stands still, and on the prism being turned still further, it begins to turn back, and therefore the angle of deviation begins to increase. The deviation, when the emerging ray becomes stationary, is then a minimum, and the angle the incident ray makes with one face of the mirror is equal to the angle the emerging ray makes with the other. When the emerging ray is in this position, note the reading on the divided circle indicated by the vernier attached to the telescope, then remove the prism, and turn the telescope so as to receive the light directly from the slit, and again note the reading. The difference between these two readings will be the angle D of minimum deviation.

Substituting its value in the formula

$$\mu = \frac{\sin \frac{D+A}{2}}{\sin \frac{A}{2}},$$

the angle A having been previously measured, μ , the index of refraction, may be found. A simple device which will enable the slit to be illuminated with monochromatic light, is to fill with a sodium salt a glass tube drawn to a fine point through which passes a fine platinum wire. If the tube be then placed so that the platinum wire is in the flame with which the slit is to be illuminated, the heated wire melts the salt,

which then runs into the flame and produces the yellow sodium light. In finding the refractive index of a liquid, place it in a glass bottle prism, and proceed just as indicated for a solid prism.

XXIII. EXAMPLES OF MAGNIFICATION WITH LENSES.

In estimating size we are guided almost wholly by the angle subtended at the eye by the object viewed. As this angle depends on the distance of the object from the observer, it is necessary to say where it, and its image formed by a lens or system of lenses, should be placed in order that we may proceed to a fit determination of the magnifying power. Common experience teaches us that the eye can accommodate itself to different distances. Persons with normal sight can see distinctly the contour and general appearance of far-removed objects, such as the sun, the moon, mountains, or distant buildings, and can also see distinctly objects placed as close to the eye as fifteen centimeters. Just as, in regard to the sense of touch, we are not able to distinguish between two points of contact, when the distance between them is less than a certain limiting value, so in the case of sight the eye is not able to distinguish points on the body viewed, when the distance between them subtends an angle at the eye less than a certain definite and determinate limit. The angle subtended by two points of course increases as they are brought closer to the observer, and hence it is that the nearer the object, the more detail there is visible. In comparing, therefore, the sizes of objects they should be placed where there is the most detail visible,—that is, at the nearest point of distinct vision,—and in considering the magnitude of an image the object should be so placed with regard to the lens that the image of it will appear to be at this distance. The smallest distance of distinct vision is, however, very different for different persons, ranging all the way from fifteen to thirty centimeters, and as the magnifying

power of an instrument should give an idea of enlargement for an eye in general, it has been agreed to take the distance of *distinct vision* as twenty-five centimeters, and in what follows we will always suppose the image, as formed by the lenses, seen there. The magnifying power of an optical instrument is generally expressed as so many *diameters*, which signifies that the linear dimensions of the image and of the object, and not their areas, are to be compared. In the case of magnifying glasses and microscopes the enlargement is taken to be the ratio of the apparent diameters of the image and of the object, both being seen at the distance of distinct vision.

The Magnifying Glass. — The simplest case of magnification is that produced by means of a single biconvex lens. In Fig. 37 let AB be such a lens, O the position of the eye, PQ the object viewed, and $P'Q'$ the image of PQ seen at the distance of distinct vision, OP' . Denoting then, OP' by Δ , OC by d , CP by p , CP' by p' , and the magnification by G , we have

$$G = \frac{P'Q}{PQ},$$

$$i.e. \quad G = \frac{p'}{p}.$$

$$\text{Now} \quad \frac{1}{p'} - \frac{1}{p} = \frac{-1}{f},$$

$$\therefore \frac{p'}{p} = 1 + \frac{p'}{f},$$

$$\text{and since} \quad p' = \Delta - d,$$

$$\text{we have} \quad G = 1 + \frac{\Delta - d}{f}; \quad (1)$$

or, if the eye is close to the lens, $G = 1 + \frac{\Delta}{f}$.

In order to test this result, take a finely divided scale for the object, and adjust the lens so that the divisions are distinctly visible. If a similar scale be placed at the distance of distinct

vision, and be viewed with one eye while the image of the second one is viewed with the other, the observer will be able, after a little practice, to tell how many divisions on the scale seen directly correspond with one division on the image. This number may be taken as a measure of the magnifying power of the lens. A number of trials should be made by placing the eye at different distances from the lens, and in order to correct any error arising from a difference in the eyes of the observer, the one should be used as often as the other in viewing each of the scales.

Doublets. — From formula (1) it is evident that the smaller we take f , the greater does the magnification become. When, however, we increase the curvature of the lens, spherical aberra-

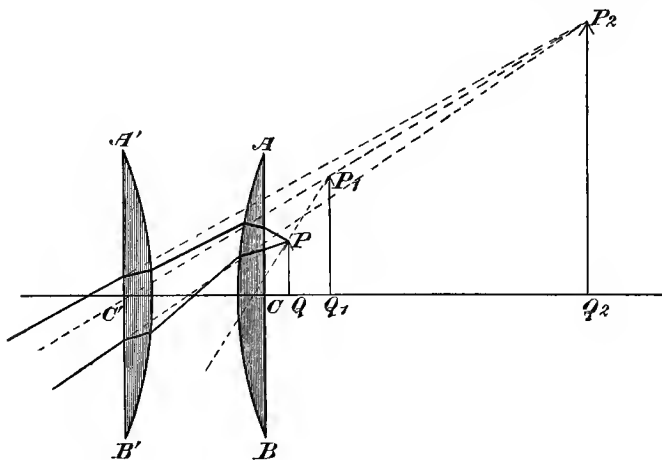


Fig. 44.

tion increases also, and in order to avoid this, and yet have considerable enlargement, it is customary to use a combination of two lenses separated by an interval, for the eyepiece of a microscope or telescope. Such a combination is called a *doublet*, and its magnifying power may be calculated in the same manner as that of a single lens.

In Fig. 44 let AB and $A'B'$ be the two lenses composing the

doublet. Let f and f' be their focal lengths respectively, and let PQ be the object viewed, P_1Q_1 its image formed by the lens AB , and P_2Q_2 the final image formed by the combination, and seen at the distance of distinct vision Δ , by an eye placed close to the lens $A'B'$. Denoting then CQ by p , CQ_1 by p' , and CC' by D , we have

$$\frac{1}{p'} - \frac{1}{p} = -\frac{1}{f}, \quad (2)$$

and

$$\frac{1}{\Delta} - \frac{1}{p' + D} = -\frac{1}{f'}. \quad (3)$$

The magnification, which is denoted by G , is given by

$$G = \frac{P_2Q_2}{PQ} = \frac{P_2Q_2}{P_1Q_1} \cdot \frac{P_1Q_1}{PQ};$$

i.e.

$$G = \frac{\Delta}{p' + D} \cdot \frac{p'}{p}.$$

Now from (3) $\frac{\Delta}{p' + D} = 1 + \frac{\Delta}{f'}$, and from (2) and (3)

$$\begin{aligned} \frac{p'}{p} &= 1 + \frac{\Delta f'}{f(\Delta + f')} - \frac{D}{f}, \\ \therefore G &= \left(1 + \frac{\Delta}{f'}\right) \left\{1 + \frac{\Delta f'}{f(\Delta + f')} - \frac{D}{f}\right\}. \end{aligned} \quad (4)$$

When the two lenses have the same focal length, and the interval between them is equal to two-thirds of this length, the doublet is called a *Ramsden eyepiece*. Its magnifying power is given by $G = \frac{1}{3} + \frac{4}{3} \frac{\Delta}{f}$. In the *Wollaston Doublet*, $f' = 3f$, D is made equal to $\frac{3}{2}f$, and its magnifying power is therefore

$$\frac{5}{6} \frac{\Delta}{f} - \frac{1}{2}$$

The combination most frequently used as the eyepiece for microscopes is that of *Huyghens*. It is sometimes called a

negative eyepiece, to distinguish it from a Ramsden or *positive eyepiece*. In it $f' = \frac{1}{3}f$, D is taken equal to $\frac{2}{3}f$, and its magnifying power is given by $G = \frac{1}{3} + 2\frac{\Delta}{f}$. When a doublet is used as an eyepiece, the lens next the objective is called the field glass, and that next the eye the eyeglass.

In testing the magnification of a doublet experimentally, exactly the same method is adopted as in the case of a single lens. If the eye is not placed close to the lens in doing this, an allowance must be made in formula (4) for its distance from it, similar to that for enlargement by a magnifying glass.

The Compound Microscope.—In its simplest form the compound microscope consists of two condensing lenses, one of which is called an object glass, or objective, and the other an eyeglass, or eyepiece. The objective, however, is usually made up of a system of lenses so constructed as to reduce chromatic and spherical aberration to a minimum, and the eyepiece most generally used is that of Huyghens. Just as in the case of doublets, an expression for magnification by a microscope can be calculated in terms of the focal lengths of the lenses, the distances these are apart, and the distance of distinct vision. Experimentally the same method as that just described for a magnifying glass may be adopted; but it is usual in the case of the microscope to modify it by placing a camera lucida over the eyepiece, and by placing a scale a little to one side of the instrument, so that, with the same eye the image of the scale under the microscope and the reflected image of the scale at the side are both visible at once. By noting how many divisions on the image formed by reflection correspond with one on that formed by refraction the magnifying power can be ascertained.

The microscope is frequently used for measuring small distances. A reference to Fig. 45 will explain how this is accomplished.

Here AB represents the objective, A_1B_1 and A_2B_2 the field

glass and eyeglass respectively of an Huyghens eyepiece; PQ the object, P_1Q_1 the image the rays go to form after passing through the objective, P_2Q_2 the real image formed by this lens, and the field glass, and P_3Q_3 the final image seen at the distance of distinct vision. A micrometer EF (consisting of a thin glass plate on which there is generally ruled a half-centimeter divided into fiftieths) is placed in the eyepiece in such a position that its image seen through the eye lens is at the distance of distinct vision. When, then, an object PQ is viewed, and its final image P_3Q_3 is seen distinctly, the *real*

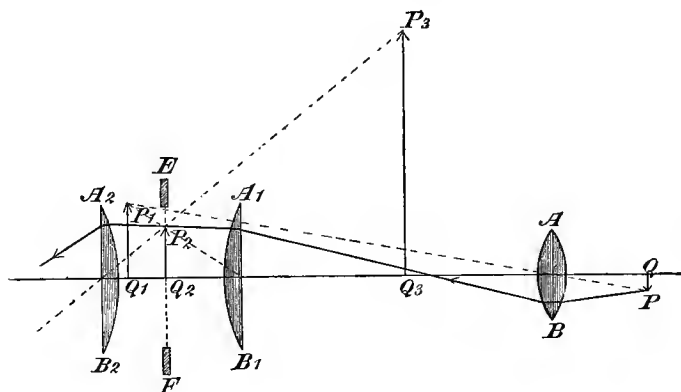


Fig. 45.

image P_2Q_2 of PQ must be in the same plane as the micrometer. If, therefore, a finely divided scale (usually called a stage micrometer) is taken for the object PQ , the observer can at once see how many divisions on this image correspond with one division on the micrometer, and so ascertain how long an object must be in order that its real image may be equal to the distance between two lines on the micrometer. The micrometer has sometimes a number of small equal squares ruled on it in place of a scale, and by means of these the number of small objects, such as colonies of bacteria, occupying a given area can be ascertained. The magnifying

power of the objective, together with the field glass, depends on the distance between them, and it should be noted that in finding the length of a small object by the method just described, this distance should be the same as in finding the number of divisions on the stage micrometer corresponding to one on that in the eyepiece.

The Telescope.—Expressions can also be calculated for the magnifying power of telescopes. In the case of the astronomical telescope it can be shewn to be the ratio $F:f$, where F is the focal length of the objective, and f that of the eyepiece. Practically the magnifying power of a telescope is determined by looking through it with both eyes open, at a distant scale, or some object marked with equal divisions, such as a picket fence. The number of divisions on the scale corresponding with one on the image is taken as a measure of the enlargement.

XXIV. EXERCISES WITH PHOTOGRAPHIC LENSES.

If the light from objects situated outside, such as trees and buildings, be allowed to enter a darkened room through a small hole in a shutter, there will be formed on a white screen placed before the hole, or on the opposite wall of a room, an inverted

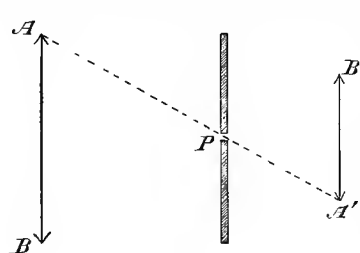


Fig. 46.

image in true colours of the outside objects. This phenomenon is due to the rectilinear propagation of light, and can be easily explained.

In Fig. 46 AB is an object situated outside, $A'B'$ is its image formed on a screen, and P is a small aperture in the shutter. A pencil of light coming from the point A will, on passing through the opening, form a spot of light at A' which cannot be distinguished from a single point if P is extremely small.

As no other point on AB can send light to A' , its colour and brightness will therefore depend on that of A , and it may thus be said to be the image of A . Similarly, B' will be the image of B , and all points between A and B will have their images between A' and B' . Hence, when light is admitted to a dark room through a small opening, we are able to form an inverted image of the outside objects. As the image of each external point is in reality a spot and not a point, it is evident that if the opening of P is large, those formed by the light coming from different points on the outside objects will overlap, and hence no image will be produced. It is for this reason, therefore, that no image is obtained when the light is allowed to enter a room through a large opening, such as a window.

The phenomenon just described affords the simplest method of taking a photograph. If the lens be removed from a camera, and a sheet of tin-foil pierced with a very fine hole be placed over the opening, an image can be formed on a sensitized plate, and a negative thus obtained. Although no "*focussing*" is required by this method, yet on account of the small amount of light forming the image, the length of time required for a proper exposure makes it generally impracticable.

The Use of the Lens. — When a lens is used, it is because it produces an image of greater intensity, and the times of exposure are therefore correspondingly diminished. Intensity, however, is then obtained at the expense of distinctness, and the images formed, owing to chromatic and spherical aberration, are far from being perfect. To remove the defects so produced, or at least reduce them to a minimum, opticians have had recourse to the use of compound lenses and of stops.

Chromatic Aberration. — When white light passes from one medium into another, the violet rays composing it undergo, owing to their greater refrangibility, a greater deviation than the red rays, and the defect caused by this in images formed with lenses is called chromatic aberration. This dispersion, or separation of the rays of different colours, varies with the two

media, and it can be readily shewn by experiments with prisms of different substances, and of different angles that, when the dispersive effects of two prisms are equal, their refractive effects are, in general, not so, and that therefore by suitably combining two prisms of different refracting angles, but having equal dispersive effects, we may produce refraction of white light without causing dispersion. It therefore follows, since by taking radial sections of a lens we may shew its general effect to be the same as that of a prism, that by suitably combining two lenses of different substances we can bring rays of light to a focus without the presence of colour effects. Hence the

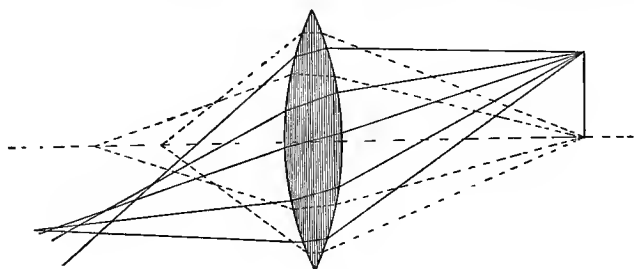


Fig. 47.

method of constructing achromatic combinations by uniting lenses made from different kinds of glass.

A simple and instructive exercise in this connection is to test the objectives of microscopes and telescopes for achromatism by allowing the light from a strongly illuminated pin-hole to pass through the combination. The emerging rays may then be examined by receiving them through a simple eyepiece, or by allowing them to form an image on a white screen.

Spherical Aberration. — When rays of light issuing from an illuminated point fall upon a convex lens, they do not, on being refracted, all intersect in a single point, but, as Fig. 47 indicates, in many of them, situated at different distances from the lens; and each of these may be taken to be the image of the point from which the rays emanate. This is well exhibited by cover-

ing the lens with a screen in which are cut a number of concentric rings. If the light be allowed to pass through only one ring at a time, it will be found that in order to focus an image the screen must be placed nearer to the lens, according as the radius of the ring through which the light passes increases.

From this it will be evident that when a convex lens is used to form an image of an object, there is in reality a multiplicity of images formed of each point; these images being situated close to each other, but yet in different planes perpendicular to the axis of the lens. When, then, an image of an object is focussed on a screen, it is in fact only so for the set of images which happens to lie in the plane of the screen. Some of the rays are focussed in front of it, and some behind it, and these on striking the screen produce a blurred appearance, and the image is thus lacking in what is technically called *definition*. This defect in images is said to be due to spherical aberration, and, in order to minimize its effects, recourse, as previously indicated, is had to the use of stops. It will be quite clear that whether these are placed in front of the lens, or behind it, their effect will be to cut off some of the rays which do not focus on the screen, or sensitized plate, and in this way produce better definition. If it were desired to produce an image free entirely from this blurred appearance, it would be necessary to use a stop with a very small opening; and although the image then formed would be sharp and distinct, the illumination would be weak, and there would be little, if any, advantage over the ordinary pin-hole method. In reducing the effects of spherical aberration stops are used which, while producing images not perfectly distinct, give moderate intensity, and so permit short exposures. For practical purposes the intensities of the image may be taken proportional to the areas of the openings in the stops, and it is an instructive exercise to calculate the relative times of exposure for a set of stops, and to test these by taking photographs of an object under constant illumination.

By placing the screen with concentric circular openings on different lenses, it may be shewn that spherical aberration depends on the curvature of the lens, and that it may be diminished by using two lenses of small, instead of one of large, curvature. These lenses can be so chosen that if they are placed a short distance apart they will have, in combination, a focal length equivalent to that of a single lens; and since by this arrangement there is a gain in definition without a counterbalancing loss in intensity, it is generally adopted in the construction of photographic objectives.

XXV. BUNSEN'S PHOTOMETER.

Although we are not able to measure absolutely the intensity of the light coming from a given source, we can, however, compare it with that from another. The eye, which cannot estimate directly, with any degree of accuracy, the relative intensity of two sources of light, is a good judge of the illuminations which they can produce; and it is on the principle of equality of illuminations that the photometers ordinarily employed are constructed.

The intensity of illumination is taken proportional to light intensity, and is defined to be the quantity of light received on a unit area of surface. The fundamental law in photometry, which is that of the inverse square, is readily demonstrated by considering a small cone of rays coming from an illuminated point. Since the same amount of light passes through any right section of this cone, the intensity at any point will vary inversely as the area of the right section containing the point; and again, as any such section varies directly as the square of its distance from the vertex of the cone, the law is evident.

In Bunsen's photometer the light from some standard, and that from the source to be tested are allowed to fall perpendicularly upon the opposite sides of a sheet of paper with an oiled spot on it. When light falls upon such a

paper, more of it is reflected, and less transmitted by the unoled portion than by the oiled, and hence the latter, when viewed from the side next the source of light, appears darker than the remainder; while if viewed from the other side, it appears brighter. If then light be allowed to fall on the two sides of the paper so that both sides are equally illuminated, that from one source, which was lost by transmission, is restored by the light transmitted from the other.

When the lights have been so adjusted that this condition obtains, the oiled part can no longer be distinguished from the rest of the paper. Let, then, d be the distance of the standard from the paper, and d' that of the light to be tested; A and A' the intensities of these two lights respectively at a unit distance, and B and B' the intensities at the paper disc.

By the law of the inverse square :

$$\frac{B}{A} = \frac{1}{d^2} \text{ and } \frac{B'}{A'} = \frac{1}{d'^2}$$

$$\therefore B = \frac{A}{d^2}, \text{ and } B' = \frac{A'}{d'^2}.$$

If, therefore, the intensities are the same at the paper, $B = B'$,

and
$$\therefore A' = \frac{d'^2}{d^2} A.$$

The intensity of the tested light is then $\frac{d'^2}{d^2}$ times that of the standard.

In making a test, different values should be given to d , and corresponding ones found for d' , the mean of the results so obtained being taken as the intensity of the light examined. As it is essential that no light should fall upon the paper disc except that coming from the two sources mentioned, the room in which the experiment is conducted should be thoroughly darkened, and its walls painted black. A simple device to permit both sides of the paper disc to be seen simultaneously

is to place it in a vertical frame, to the back of which are attached two mirrors inclined at an angle to the paper such that the observer can by looking into the mirrors, see images of both sides of the paper at the same time. The oiled spot is generally made circular; but it is well to check results by making it of some irregular form. A modification of this is obtained by placing together three sheets of paper, in the center one of which is a hole of any desired shape. Owing to the presence of light reflected from the walls of the room, and to unequal absorption by the two parts of the disc, it is rarely possible to arrange the two lights so that the oiled spot entirely disappears. Indeed, the results obtained by the method are only approximate at best. It is assumed in the theory that the sources of light are points, and that the lights are of the same quality, — conditions which do not hold in practice.

Various forms of standard lights have been devised; but what seems to give the steadiest light is to illuminate a small hole in a screen by means of the flame of a coal-oil lamp burning a broad wick. As, in testing gas flames and incandescent lamps, it is found that the intensity at a given distance varies with the position of the flame in the one case, and with that of the carbon filament in the other, the student should make a table of his results, or plot a curve shewing the intensity of the light in different positions. Bunsen's photometer may also be used to compare the absorptive powers of different transparent media, such as glass, or water, by finding the intensities of the light transmitted by them from a given source. The results obtained, however, will not be exact, as light is always reflected from the surface of the medium employed.

XXVI. AYRTON'S PHOTOMETER — MODIFIED FROM BUNSEN'S.

When the light to be examined is very great compared with that of the standard, the experiment takes up so much room, if the previous method is adopted, that it is quite impracticable.

This difficulty has been overcome by Ayrton, who introduces a biconcave lens between the light tested and the photometer.

Theory. — In Fig. 48 P is the source of the light to be tested, AB the lens, ST the paper disc of the photometer, and CPD a cone of light striking the lens. Let A and A' be the intensities of the standard and of the given light at a unit distance; B and B' the intensities of the light from the latter at the disc, with the lens interposed, and with it removed respectively, and B'' that of the standard at the disc. Also let C be the area of the circle of light on the disc formed by the cone of rays DPC with the lens inserted, and C' with it removed;

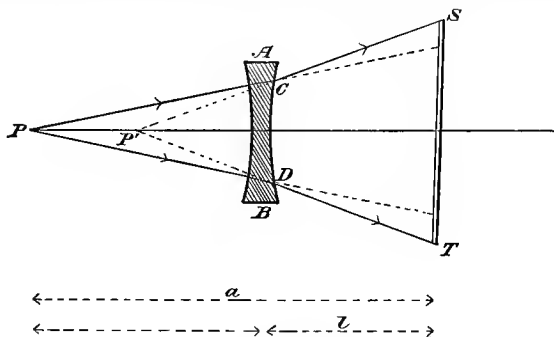


Fig. 48.

and let a be the distance from the paper disc to the light tested, c that to the standard, and l that to the lens.

Since the intensity of illumination is the amount of light on a unit area, and since the total amount of light on C and C' , under the conditions mentioned above, may be taken to be the same, we have

$$BC = B'C', \text{ or } B' = \frac{C}{C'}B.$$

Again, from the fundamental law we have

$$\frac{A'}{B'} = a^2, \text{ or } A' = a^2 B';$$

i.e.

$$A' = a^2 \frac{C}{C'} B, \text{ or } B = \frac{A' C'}{a^2 C}.$$

Again, we have $\frac{B''}{A} = \frac{1}{c^2}$ or $B'' = \frac{A}{c^2}$.

Therefore when the two sides of the disc are equally illuminated, we have $B' = B$,

or
$$\frac{A'C'}{a^2C} = \frac{A}{c^2}, \text{ i.e. } \frac{A'}{A} = \frac{a^2C}{c^2C'}$$

Now denote the circular area of the lens through which the light passes by C'' , and let the light after refraction appear to come from a point P' at a distance x behind the lens.

Then
$$\frac{C'}{C''} = \frac{a^2}{(a-l)^2}, \quad (1)$$

and
$$\frac{C}{C''} = \frac{(l+x)^2}{x^2}. \quad (2)$$

Also, since P' is the image of P ,

$$\begin{aligned} \frac{1}{x} - \frac{1}{a-l} &= \frac{1}{f}, \\ \therefore x &= \frac{f(a-l)}{f+a-l} \end{aligned} \quad (3)$$

From (1), (2), and (3),

$$\begin{aligned} \frac{C}{C'} &= \frac{[l(a-l) + fa]^2}{a^2f^2}, \\ \text{and } \therefore \frac{A'}{A} &= \frac{a^2}{c^2} \frac{C}{C'} = \left\{ \frac{l(a-l) + fa}{cf} \right\}^2; \end{aligned} \quad (4)$$

i.e. the intensity of the given light will be

$$\left\{ \frac{l(a-l) + fa}{cf} \right\}^2$$

times the standard.

Experiment. — Place the biconcave lens in a screen, and insert it between the light to be tested and the paper disc of the photometer. Then, as in the previous method, adjust the standard so that the oiled spot cannot be distinguished from the rest of the paper, and measure the lengths a , c , and l .

By substituting in the formula these values together with that of f , the focal length of the lens, which may be found by either of the methods given in Experiment XXI., the intensity of the light may be determined. In making a test, the student should obtain a number of results by giving different values to a and l , and therefore to c .

Ayrton's method is especially applicable in finding the intensity of the light from an incandescent lamp, or that from an electric arc. The intensity of the latter should be examined with the carbons making different angles with the vertical, because, just as in the case of gas flames and incandescent lamps, the intensity at a given distance varies with the position of the arc.

XXVII. ROTATING DISC PHOTOMETER.

When a number of objects are passed in rapid succession before the eye of an observer, the effect, owing to the persist-

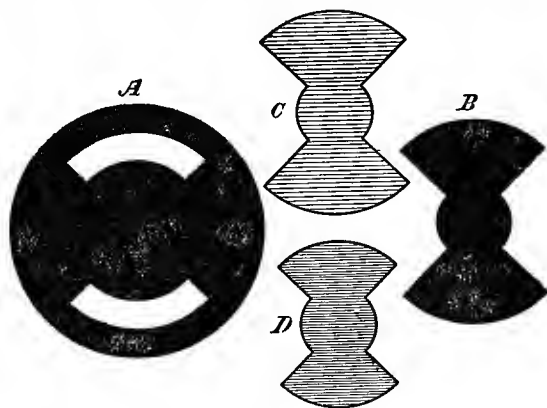


Fig. 49.

ence of luminous sensations, is the same as if the retina received the impressions from the different objects simultaneously; and it is for this reason that we are able to produce a white appear-

ance by the rapid rotation of a disc of coloured sectors arranged in proper proportions, or by rapidly oscillating the spectrum produced by allowing a pencil of the sun's rays to fall on a prism. The principle here involved has been applied with considerable success to photometric investigations. In Fig. 49, *A*, a black circular disc made of thin sheet metal, or of cardboard, has two apertures in it, whose sides are circular arcs concentric with the disc; *B*, a double sector of the same material as the disc, is used to reduce, or enlarge the size of these apertures; and *C*, *D*, two double sectors of some gray-coloured material, are of such a size that when they are placed concentric with the disc *A* the lighter one *C* extends just beyond the outer edges of the apertures, while the darker one *D* extends half-way across them.

The fundamental hypothesis in the work with rotating discs is that the amount of light coming from a sector is proportional to the angle of this sector. The edges of the plates used play a considerable part in these investigations, and experiment seems to show that, if the angles are not small, and the edges of the plates are made thin by bevelling, the assumption is warrantable.

EXERCISE I. — *Comparison of gray tints.*

It is often desirable to have an accurate notion of the relative quantities of light reflected from different papers commonly called white. In order to investigate this, cut two sectors such as *C* and *D* from the two sheets of paper to be examined, and place them on the same axis of rotation as the black disc *A*, and behind it, the darker sector being next to the disc. If, then, this arrangement be rotated in front of a black screen of velvet, or other such material, there will appear two concentric rings of a drab colour produced by the combination of the white of the sector with the black of the disc. As their colour depends only on the amount of the white in combination with the black, the two rings may be made of the same tint by taking suitable angles for the sectors of paper. If great accuracy is desired, the aux-

iliary sector B may be used. If, for example, it is found that the black of the disc combined with 81° of the white of one sector gives nearly the same tint as when combined with 80° of that of the other, then, by adding 1° to each of the sectors, or by taking 1° off, we may get the ratios $82:81$ or $80:79$, one of which will be a closer approximation to the exact result. If a and b are the angles of the two white sectors, when the rings are of the same tint, and Q, Q' are the quantities of light reflected from a unit angle of each of these sectors, then $aQ = bQ'$, or the ratio $b:a$ is a measure of the relative whiteness of the two papers. If the edges of the apertures are divided into degrees and parts of a degree, the angles a and b may be read directly. This same method may be applied to a study of the light reflected from walls of different gray tints. It is done by spreading thin layers of the plasters used on different sectors, and then, when it is dry, proceeding just as in the paper tests. In conducting these experiments it can be seen that the colour of the two rings alters slightly with the direction of the incident light, and with the position of the observer. It will be found that the best results are obtained when the disc is illuminated with rays of light incident parallel to the axis of rotation, and when a small part only of the rings is viewed by means of a telescope.

EXERCISE II. — *To compare the intensities of light coming from two independent sources.*

To perform this experiment the disc should have two concentric sets of apertures a short distance apart, and the light should be allowed to fall on the back of the disc in such a manner that each of the sets is constantly illuminated by the light from one of the two sources. By altering the size of these apertures until the colours of the rings, formed by the rotation of the disc, are the same, a measure of the intensities of the light from the two sources may be obtained. If a and b are the angular openings, the intensities are then in the ratio $b:a$.

As considerable difficulty is likely to be met with in arranging suitable mechanism for this method, a better one is to use two small discs with a single set of apertures in each, and to rotate them in the same plane. In this way the illuminations may be made more simply, and more nearly under the same conditions. It is evident that these methods only apply when the lights examined are of the same quality. Whenever the question of colour comes in, the investigation becomes complicated and uncertain.

EXERCISE III. — *To compare the absorptive powers of different media.*

If a screen be made partly of one translucent substance, and partly of another, and light from a single source be allowed to fall on the back of this screen, then that which comes through may be considered as coming from two independent sources. The intensity of the light coming from each part of the screen may then be investigated by either of the methods outlined in Exercise II., and the absorptive powers of the different substances composing the screen compared.

In Exercises II. and III. every precaution should be taken to insure that no light falls on the apertures of the discs except that which is being examined.

XXVIII. SPECIFIC HEAT OF SOLIDS.

The Method of Mixture. — In Fig. 50 *B* is a boiler, in which water may be heated, and *AC* is a steam jacket connected to it by a tube indicated in the figure. To the upper portion of the tube *DB*, leading from the boiler to the outside air, there is fitted a water jacket through which cold water is kept constantly running. The calorimeter *G* consists of two metallic vessels, generally made of sheet brass, the one resting on wooden supports inside the other. The solid *E*, whose specific heat is required, is suspended by a string in the inclosure *F* within the

steam jacket. The steam after leaving the boiler passes into this jacket, and then out into the tube DB , where, on coming into contact with the cold air, it condenses and falls back into the boiler. In this way a steady supply of steam is led into the jacket, whose temperature is thus kept constant, and there is, therefore, no occasion for renewing the water in the boiler. The steam jacket is generally provided with attachments by means

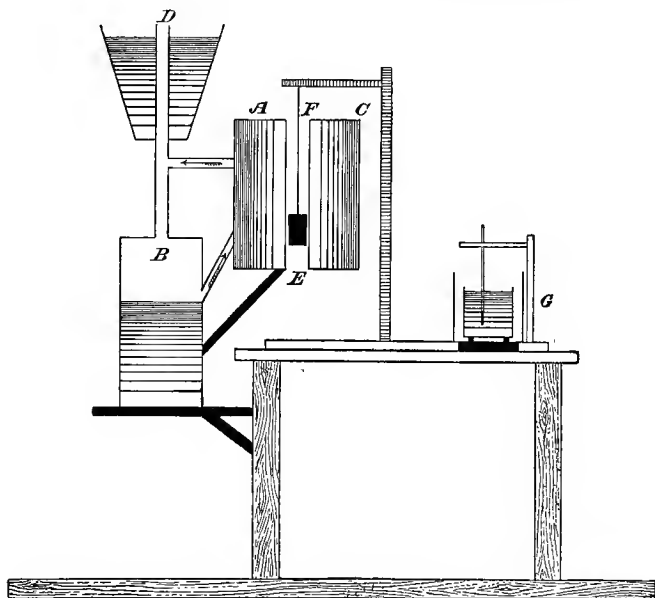


Fig. 50.

of which both ends of the inclosure F may be stopped, and radiation thus prevented.

The unit of heat generally adopted is the quantity required to raise the temperature of one gram of water one degree centigrade, and is called a *calorie*. It is found that, if equal masses of two different substances are subjected to the same heat, under the same circumstances, for a given time, their temperatures will vary considerably.

This shews that different quantities of heat must be imparted to equal masses of different substances to make the same alteration in their temperatures, and the *specific heat* of a substance is defined to be the ratio of the quantity of heat required to raise the temperature of a given mass of this substance one degree, to that required to raise the temperature of an equal mass of water one degree.

Theory. — Let M be the mass of the solid in grams, T° its temperature before it is dropped into the water; m and m' the masses of the water in the calorimeter, and of the calorimeter respectively; t° and θ° their initial and final temperatures, and c , and c' the specific heats, respectively, of the given solid, and of the substance of which the calorimeter is made.

The heat given up by the solid will then be equal to $cM(T - \theta)$ calories, while that gained by the water will be equal to $m(\theta - t)$ calories, and that by the calorimeter $m'c'(\theta - t)$. Since the heat lost by the solid is gained by the calorimeter and the water it contains, we have

$$cM(T - \theta) = m(\theta - t) + m'c'(\theta - t),$$

or
$$c = \frac{(m + m'c')(\theta - t)}{M(T - \theta)}.$$

Water Equivalent of the Calorimeter. — The expression $m'c'$ in this formula is called the water equivalent of the calorimeter, since from its position it is equivalent to an addition to the mass of water in the calorimeter. Its value is best obtained by finding its mass m' by weighing, and by taking the specific heat c' of the substance of which it is made, from the tables, but it may also be found experimentally in the following manner: Into a mass M_1 of water contained in the calorimeter (the temperature of both being t_1°) pour a mass of water M_2 of temperature t_2° , higher than t_1° , and let θ_1° be the result-

ing temperature. By the same process of reasoning as above we will have

$$M_2(t_2 - \theta_1) = M_1(\theta_1 - t_1) + m'c'(\theta_1 - t_1),$$

$$\therefore m'c' = \frac{M_2(t_2 - \theta_1)}{\theta_1 - t_1} - M_1;$$

i.e. $m'c'$ is expressed in terms of quantities which can be directly found.

Experiment.—Weigh the solid E whose specific heat is to be determined, and then suspend it together with a thermometer in the inclosure within the steam jacket, as indicated in the figure. Care should be taken to keep a constant stream of cold water running through the vessel attached to the upper part of the tube DB , and not to fill the boiler with water higher than the opening in the tube leading from it to the steam jacket. While the solid is being heated to the temperature of the steam, the student may find the water equivalent of the calorimeter experimentally. In conducting this preliminary part of the experiment care should be taken to have the temperatures, and the quantities of water involved as nearly as possible the same as those in the experiment proper. Weigh the calorimeter, including thermometer and stirrer. Partly fill it with water of temperature t_1° , below that of the room, and again weigh it. The difference between these two weights will give the mass of water M_1 in it. Then pour into the water M_1 a sufficient quantity of water of temperature t_2° to bring the resultant temperature as much above that of the room as t_1° was below it. In this way the loss of heat due to radiation may be partly overcome. Again weigh the calorimeter, and the difference between this weight and the last will give the mass M_2 of the water poured in. The quantities t_1 , t_2 , θ_1 , M_1 , M_2 thus being known, the water equivalent may be determined as previously indicated. The vessel from which the water M_2 is poured should contain more than is actually necessary, and t_2

should be the mean of the temperatures of this water before and after M_2 is poured out. Especial care should be taken to pour the water into the calorimeter in such a manner that the least possible amount of heat is lost in the pouring.

Having determined the water equivalent, again take the calorimeter, and partly fill it with water of temperature t° and weigh it. The difference between this weight and that of the calorimeter empty will give the mass m of the water poured into it. Having made certain that the solid is now at the temperature of the steam jacket T° , which is noted by reading the thermometer suspended within the inclosure F , or by finding from tables the boiling-point of water corresponding to the reading of a barometer suspended in the room, slide the calorimeter under the steam jacket. Then drop the solid into it, and stir the water gently. The temperature of the water will rise gradually, then remain stationary for a time, and finally begin to lower. The stationary temperature θ° will then be that resulting from plunging the heated solid into the water. The mass M of the solid having been previously found, and m , $m'c'$, θ , t , and T thus being known, we can, by applying the formula, obtain the specific heat of the solid.

XXIX. SPECIFIC HEAT OF LIQUIDS.

In Fig. 51, AB is a boiler containing water, and CD is a hollow, metallic reservoir almost wholly immersed in it. The calorimeter consists of three parts, EF resting on a stand, GH resting on wooden supports inside of EF , and K immersed in water contained in GH . The different parts of the calorimeter are generally made of thin brass, the part K being made of such a form as to expose as much surface as possible to the water. As the figure indicates, it is connected to the reservoir by a tube in which there is a tap.

Theory.—In this case, also, the method is one of mixture. The liquid whose specific heat is to be determined, is first heated

in the reservoir CD , and then allowed to run into the calorimeter, where it gives up its heat to the water, and the vessel containing it. Let M_1 , M_2 , and M be the masses, in grams, of the calorimeter, the water in it, and the given liquid respectively, t_1° and θ° the initial and final temperatures of the water in the calorimeter, and t° the temperature of the liquid before it is run into the calorimeter. Also let c and c_1 be the specific

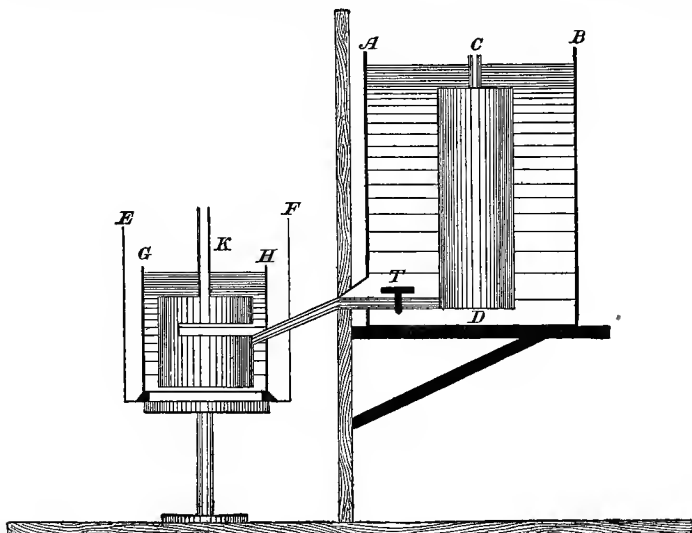


Fig. 51.

heats of the given liquid and of the metal of which the calorimeter is made, respectively.

The heat given out by the liquid, on being run into the calorimeter, is then equal to $cM(t-\theta)$ calories, while that gained by the calorimeter is $c_1M_1(\theta-t_1)$, and that by the water in it $M_2(\theta-t_1)$.

$$\therefore cM(t-\theta) = (M_2 + c_1M_1)(\theta-t_1),$$

or

$$c = \frac{M_2 + c_1M_1}{M} \left\{ \frac{(\theta-t_1)}{(t-\theta)} \right\}.$$

Water Equivalent of the Calorimeter.—In this experiment, as in the last, c_1M_1 is the water equivalent of the calorimeter, and may be found by either of the methods there given. If the experimental method is adopted, the error in this case arising from the water cooling when being poured into the calorimeter, may be avoided by heating it in the reservoir CD , and then allowing it to run through the tube connecting the two vessels.

Experiment.—First weigh the two parts of the calorimeter GH and K , together with the stirrer and a thermometer. Then pour in water sufficient to cover the part K , as indicated in the figure, and again weigh them. The difference between the two weights will give the mass, M_2 , of the water poured in. In the mean time, suspend a thermometer in the boiler, and place in the reservoir CD enough of the liquid whose specific heat is to be determined to nearly fill the vessel K , and allow it to become heated by the surrounding water.

In order to prevent evaporation, care should be taken to keep the top of the reservoir closed during the heating process, and not to let its temperature rise above the boiling-point of the given liquid corresponding to atmospheric pressure. When it has been sufficiently heated, the temperature t° of this liquid may be noted by reading the thermometer suspended in the water. Also note the temperature t_1° of the water in the calorimeter, and then turn the tap, and allow the liquid to run into it through the tube.

Gently stir the water in the calorimeter, and when its temperature has become stationary, note its value θ° . Then weigh the calorimeter and contents, and the difference between this weight and the last will give the mass M of the liquid that ran in. This result may be checked by first weighing the liquid before it is poured into the reservoir. The student will thus see that the only essential difference between this experiment, and the last is that the substance whose specific heat is desired,

is not in this case allowed to come in contact with the water in the calorimeter.

Especial care should be taken to choose conditions favourable to overcoming the errors due to evaporation, as in this experiment considerable inaccuracies of this kind are almost certain to be met with by the student.

The specific heat of a given liquid may also be calculated by finding the specific heat of a solid relative to it, and also relative to water. If C_1 and C_2 are these two specific heats respectively, and C the specific heat of the liquid, we have

$$C_1 C = C_2,$$

or
$$C = \frac{C_2}{C_1}.$$

XXX. LATENT HEAT OF FUSION OF ICE.

If a certain mass of water at 0°C. is placed in one beaker, and an equal mass of ice at 0°C. in another, and if the two are then placed in a hot-water bath, it is found that at the instant the ice is all melted the temperature of the water produced by it is still 0°C. , while that of the water in the other beaker is about 80°C. From the similarity of the conditions to which the two vessels are subjected, it is evident that the same quantity of heat passes into each during the time they are in the hot water. In the one case the heat is used up in raising the temperature of a substance, while in the other it is used up in changing the physical structure of the substance without altering its temperature. From this fact the heat absorbed in the latter case has been called *latent heat*, and technically the *latent heat of fusion* of a substance is defined to be the number of units of heat required to convert a unit mass of it from the solid to the liquid state without raising its temperature. Adopting the units of heat indicated in Experiment XXVIII., the latent heat of fusion of ice is the number of calories required to con-

vert one gram of ice at 0° C. into water without altering its temperature.

Theory. — Into a quantity of water M contained in a calorimeter of mass M_1 , drop a mass of ice M_2 of temperature 0° C. Let t_1° and θ° be the initial and final temperatures of the water respectively, L the latent heat of fusion of ice, and c_1 the specific heat of the substance of which the calorimeter is made. LM_2 calories is then the quantity of heat absorbed in melting the ice, and $M_2\theta$ calories that in raising the temperature of the water produced by it from 0° C. to θ° C. Since the quantity of heat given up by the calorimeter and the water in it is $(M+c_1M_1)(t_1-\theta)$, we have

$$(M+c_1M_1)(t_1-\theta)=LM_2+M_2\theta,$$

or

$$L=\frac{(M+c_1M_1)(t_1-\theta)}{M_2}-\theta.$$

Experiment. — The masses M , M_1 , and M_2 are found by weighing, and just as in the two previous experiments, the water equivalent of the calorimeter c_1M_1 may be determined either by a reference to a table of specific heats, or by the experimental method. The initial temperature t_1° , and the masses M and M_2 of the water and ice respectively, should be so selected that the final temperature θ° is as much below the temperature of the room as t_1° is above it. These quantities may be ascertained approximately by a calculation, but the student will find it exceedingly instructive to adopt a tentative process, and in this way arrive at the proper proportions that should obtain in an exact determination of L . As it is essential that the ice should melt as rapidly as possible, it is best to break it up into small pieces, and to dry each piece thoroughly with blotting paper before it is placed in the calorimeter. Care should also be taken to have the temperature of the ice at 0° C., by placing it in a warm room a short time before the experiment is commenced.

XXXI. LEVEL TESTING.

The spirit level is made of a glass tube slightly curved, so that its axis forms part of a circle of large radius. Usually when tubes come from the manufacturer they have a slight curvature at each point, and it is only necessary to perfect this in the piece chosen by grinding the inside with emery powder. The tube is filled with alcohol, or ether, excepting a small space containing a bubble of air which tends to occupy the highest part of the tube. It is sealed as shewn in Fig. 52, and then firmly cemented in a brass tube, which is itself attached to a brass plate (Fig. 53) in such a manner that when the plate rests on a level surface the air bubble remains stationary at the center of the opening in the upper side of the brass tube. In



Fig. 52.



Fig. 53.

some levels a thin brass strip across the opening marks this position of the bubble; while in others its position is marked by the central division of a scale ruled on the glass.

For fine work, a level should be very sensitive, and should be ground to a true curvature, which is indicated by a uniform run of the bubble when it is given a slight inclination to the horizon. As the sensitiveness of a level should be in strict keeping with the instrument to which it is attached, it is necessary that each one should be thoroughly tested before being used for any particular work.

Buff and Berger's Level Trier, an instrument for this purpose, is shewn in Fig. 54. It consists of an iron plate upon which is mounted an iron bar having at one end two pivotal centers resting in receptacles provided for them in the base plate, and at the other a micrometer screw carrying a graduated disc. The bar is provided with fixed wyes in which levels to

be tested may be placed, and also with a scale, to be used in case none is ruled on the level. The points of the pivots and of the micrometer screw are made very hard, and the latter bears on a plate with a hard and polished surface. This plate is often made to move eccentrically with regard to the screw, so that the point of rest can be changed in case of wear.

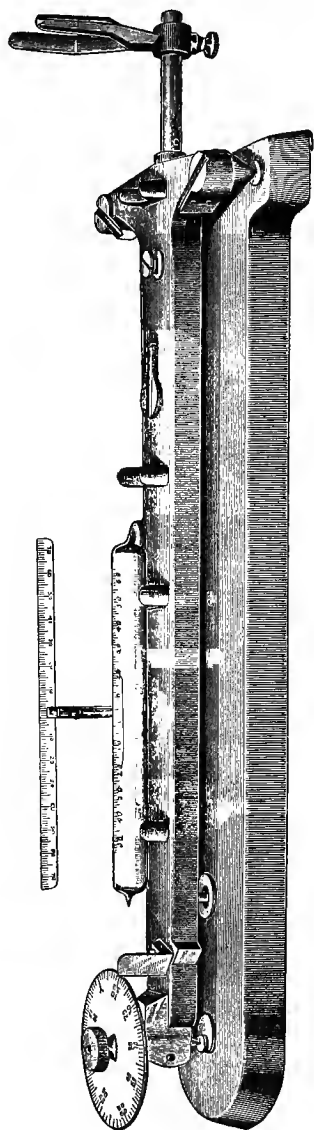


Fig. 54.

Theory. — Let x be the height through which the left-hand side of the instrument is raised on turning the micrometer screw, and let y be the distance between the screw point and a point midway between the two pivots. The circular measure of the angle through which the instrument has been turned is then $\frac{x}{y}$, and if this ratio is multiplied by 3437.748 or 206264.9, the angle is given in minutes or seconds respectively.

EXERCISE I. — *To investigate the run of the bubble.*

Place the level to be tested on the wyes on the bar, and then raise, or lower the latter until the bubble is at one end of the scale, or, if the level has no scale, at the point which is the

limit of the run of the bubble in practice. The micrometer disc is then turned over equal spaces, and the run of the bubble carefully noted. When the bubble has been moved over its course, it should be moved in the opposite direction in the same manner, and the whole operation repeated several times. The mean value of all the observations may then be determined, and the value of one division on the level expressed in minutes or seconds of an arc, as the case may be. If the level to be tested cannot be easily removed, the entire instrument to which it is attached may be placed on the trier, resting on points provided for it directly over the pivots.

In the following table, exhibiting the manner in which a test was made, A and B are the right and left hand sides of the air bubble respectively :

Number of Trials.	Micrometer Readings.	Level Scale Readings.		Differences.		Length of Bubble.
		A End.	B End.	A End.	B End.	
1	7	9.8	51.8	4.4	4.6	61.6
2	17	14.2	47.2	4.3	4.5	61.4
3	27	18.5	42.7	4.5	4.2	61.2
4	37	23.0	38.5	4.0	4.1	61.5
5	47	27.0	34.4	4.5	4.4	61.4
6	57	31.5	30.0	4.3	4.3	61.5
7	67	35.8	25.7	4.4	4.7	61.5
8	77	40.2	21.0			61.2
8	77	40.0	21.2	4.5	4.3	61.2
7	67	35.5	25.5	4.4	4.7	61.0
6	57	31.1	30.2	4.4	4.4	61.3
5	47	26.7	34.6	4.4	4.2	61.3
4	37	22.3	38.8	4.1	4.0	61.1
3	27	18.2	42.8	4.2	4.3	61.0
2	17	14.0	47.1	4.5	4.4	61.1
1	7	9.5	51.5	60.9	61.1	61.0

Mean value of differences, 4.36 twentieths of an inch.

The pitch of the micrometer screw in the instrument used was one-sixtieth of an inch, its disc divided into one hundred

parts, the level scale graduated to twentieths of an inch, reading right and left from zero at the center, and the distance between the micrometer-screw point and the line joining the two pivots was 17.9 inches. By a simple calculation it may be seen that the angle corresponding to a rotation of 10 divisions on the micrometer disc is 19.2 seconds, and that therefore an inclination of 4.4 seconds caused the bubble in the level tested to move over one-twentieth of an inch. This then may be taken as an indication of the sensitiveness of the level. The uniformity exhibited in the displacements of the bubble shews that the curvature of the level was very regular.

EXERCISE II. — *To find the radius of curvature of a level.*

This is found by rotating the level through some known angle, and noting the displacement of the bubble produced. If z is this displacement, and $\frac{x}{y}$ the circular measure of the angle of rotation, the radius of curvature of the level is given by

$$r = \frac{zy}{x}.$$

A great source of error in spirit levels, increasing with their sensitiveness, is an unequal heating of the level tube. The bubble will always move towards the warmer spot, or end, on account of a changed condition in the adhesiveness of the fluid, and so a spirit level, while being tested, should never be touched by the fingers, nor breathed upon, and it should always be protected from the heat of the sun, or of artificial lights. Care should also be taken to allow sufficient time for the bubble to settle before taking a reading.

PART II.

ADVANCED COURSE.

ACOUSTICS, HEAT, ELECTRICITY AND MAGNETISM, WITH
AN APPENDIX ON THE DETERMINATION OF
GRAVITY, AND ON THE TORSION
PENDULUM.

ACOUSTICS.



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EXPERIMENTS.

I. THE SONOMETER.

THIS is a large sounding box, a little more than a meter in length, provided with a bridge at each end, over which eight strings are stretched, either by means of weights, or by fixed pins and thumbscrews, to which the strings are attached. The distance between the bridges is usually one meter, and a wooden scale is provided alongside, divided into millimeters. Clamps with weights are also provided, or else sliding bridges, to enable one to set any desired portion of a string in vibration. The method of using the instrument is to obtain steel strings about half a millimeter in diameter, and, stretching them over the bridges, to tune them all in unison with any chosen pitch. This may be done very accurately (even by what is termed an unmusical ear) after a little practice, by listening for the *beats* which are produced when two sets of waves, differing slightly in their periodic times, are sent from two separate sources simultaneously to the ear. The tension of any particular string is increased or diminished until it begins to beat with the one chosen as a standard ; the beats will be at first very rapid, and cause a peculiar rolling effect in the ear, which must be heard to be appreciated ; as the tension is properly altered they become slower and slower, and finally disappear. The two strings are then in unison, and the same process is repeated with the others, until all give the same note. In order to vibrate a string properly, it should be plucked with the thumb and forefinger at its center ; or, better still, the ball of the thumb is placed on the

middle point of the string, the direction of the thumb being nearly at right angles to the string, and then, with a light pressure, it is allowed to slip off, thus producing the fundamental note, comparatively free from the higher harmonics. Steel strings are preferable to others, as they can be obtained almost perfectly uniform throughout, give a loud tone, and last indefinitely; but they must be plucked properly, or else they give most disagreeable harmonics, which may interfere with tuning. When all the strings are in unison, the *major diatonic* scale may be formed by choosing the first open string as the fundamental or starting note, and then adjusting the clamps with weights on the other strings, so that the lengths put in vibration are to the length of the fundamental in the ratios :

$$\frac{8}{9}, \quad \frac{4}{5}, \quad \frac{3}{4}, \quad \frac{2}{3}, \quad \frac{3}{5}, \quad \frac{8}{15}, \quad \frac{1}{2}.$$

This simply assumes the definition of the major diatonic scale to be one whose notes are as the ratios :

$$1, \quad \frac{9}{8}, \quad \frac{5}{4}, \quad \frac{4}{3}, \quad \frac{3}{2}, \quad \frac{5}{3}, \quad \frac{15}{8}, \quad 2;$$

and also that the pitch of a string varies inversely as its length. The *scale of equal temperament* may be obtained for any open string by taking the other lengths in the ratios :

$$\frac{1}{\sqrt[12]{2}}, \quad \frac{1}{\sqrt[12]{4}}, \quad \dots \quad \frac{1}{2}.$$

The ordinary *Minor* and *Harmonic Minor* scales may be had by choosing lengths similar to the *major diatonic*; except that for the former the ratio $\frac{4}{5}$ is made $\frac{4}{5} \times \frac{25}{24} = \frac{5}{6}$, a *minor third*; and for the latter $\frac{4}{5}$ is made $\frac{5}{6}$, a *minor third*, while $\frac{3}{5}$ is made $\frac{3}{5} \times \frac{25}{24} = \frac{5}{8}$, a *minor sixth*.

The *harmonic scale* is formed by taking lengths on a stretched string inversely proportional to the natural numbers 1, 2, 3, 4, ... and so on.

By listening carefully a great many of the notes of this scale can be heard in the case of any freely vibrating string, their

presence there being theoretically established by what is known as *Fourier's* theorem.

The notes of this scale, when heard as auxiliaries to the fundamental note of a string, are usually called *harmonics*, and are of the greatest importance in the theory of Music.

The formation of the scales is based upon the assumption that the strings are all of the same material, and uniform throughout; and, the tension of all being the same, the pitch of any portion of a string depends inversely on its length. See Exp. 3.

II. THE SCALE OF EQUAL TEMPERAMENT.

The accurate scale of physicists, determined by the series of vibration, ratios,

$$1, \frac{9}{8}, \frac{5}{4}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{15}{8}, 2,$$

is seldom used for musical purposes, on account of the difficulties which have been met with in arranging mechanism for instruments to play in different keys. A musical instrument like the piano, which would play accurately, according to the true scale of the physicist, in all keys, would require more than twenty-five notes to the octave. This can easily be seen by writing down all the notes necessary for playing in the various keys, and then striking out those which are repeated. Thus, if we commence at middle *C* on the piano, we can obtain the true major scale by tuning eight strings, according to the ratios expressing pitch. If we did this throughout the keyboard, we should have a complete major scale (all white notes) in perfect order. But suppose now we wish to play perfect intervals in the key of *D*: it is evident we must introduce new notes which may not correspond at all with those in the key of *C*; and as we proceed from one key to another we find additional notes necessary for each change of key. And then, if we take account of flats and sharps, we are introducing fresh difficulties.

Many attempts were made to overcome this mechanical difficulty by reducing the number of notes on the keyboard; but the only method which allowed playing in all the keys was the one finally adopted. In the *scale of equal temperament* the octave was defined to be twelve semitones; and, as the pitch of the octave is twice that of the fundamental, a scale of thirteen notes in the octave was chosen, with twelve intervals, the pitch of each being obtained from the next lower by multiplying by the twelfth root of two, which corresponds to the interval called a *semitone*.

In most stringed instruments, the scales are approximately scales of equal temperament, although in the case of the piano more latitude is allowed the tuner, who generally makes some intervals a little greater than others, being guided more by his individual taste than by the factor which is represented mathematically by $\sqrt[12]{2}$.

One may easily construct a scale of equal temperament for any fundamental *C* note by using a *tonometer* or series of forks, differing from one another by 8 or 16 vibrations per second. The set used for experimental purposes generally commences with 256 or 512 v.s.,* and runs up to 1024. For example, to construct the tempered scale commencing with 512, one would calculate the vibration numbers corresponding to the other twelve notes by using the factor $\sqrt[12]{2}$. The complete scale would then be given by the numbers, 512, 542.4, 574.7, 608.9, 645.1, 683.4, 724.1, 767.1, 812.7, 861.1, 912.3, 966.5, 1024.

To obtain the vibration number 542.4, two forks of the *tonometer* are chosen, 536 and 544, and these, if put in vibration together, give four beats per second; if now the pitch of 544 be

* *Note on Pitch Notation.* — Confusion often arises in the calculation of pitch from the fact that the English use the term *vibration* in the same sense as the complete vibration of a pendulum, *to and fro*, while the French System takes it to be a *simple* vibration; so that a French fork numbered 512 would be the same as an English 256. The difficulty is overcome by marking v.s., or v.d., after the pitch, as is done by König on his standard forks, to indicate *vibrations simples* (simple vibrations), and *vibrations doubles* (complete vibrations). We have adhered to this in the present course.

altered by placing small pellets of wax on the prongs so that it moves more slowly, it is evident that its pitch can be reduced to the required number, 542.4. In other words, when the loaded fork gives, with 536, 3.2 beats per second, or 32 beats in 10 seconds, then the former is vibrating 542.4 times per second. The process then becomes one of counting beats with a clock or stop watch, and loading the forks with wax until the necessary numbers of beats are obtained in each case.

The following precautions should be taken :

1. When vibrating, the prongs of a tuning fork move out and in, together : so, if one sets a fork in vibration with a bow, it should be drawn across the upper end of *one prong only*, and in such a way that the plane of the horse hair is nearly in the plane containing the two ends of the prongs. If inclined at an angle, a pure note is not obtained, and the horse hair is very liable to get torn. The fork may be put in vibration by bowing at the side of the prongs ; but in that case the fundamental tone of the fork is often accompanied by a disagreeable high note. See Exp. 7 (3).

2. In counting the beats, the beginner is very apt to count the maximum from which he starts as one : a little practice soon enables him to avoid this.

As large a number as possible should be counted in every case to minimize the error ; usually the counting can be carried on for thirty or forty seconds, except in the case of high forks which have small amplitudes and soon stop vibrating.

3. In loading the forks, when very slight differences are desired, it will be found more convenient to move the pellets of wax up or down the prongs, instead of increasing or diminishing the loads themselves.

4. Tuning forks are always made of steel and change rapidly if rusted. Care must then be always taken of them by the experimenter ; and when an experiment is finished they should be carefully cleaned with a soft rag moistened with alcohol, dried, and put away in a dry place.

III. LAWS OF THE TRANSVERSE VIBRATIONS OF STRINGS.

If a string of any material be stretched beyond its natural length between two fixed points and then be disturbed from its position of equilibrium, it vibrates to and fro, and emits certain musical notes, of which one is much more prominent than any other: this is the lowest or *fundamental note* of the open string, and its pitch or vibration number is usually spoken of as the *pitch of the string*. This pitch will evidently depend on the length of the string, its diameter, tension, and the material of which it is made; and a relation connecting these elements can be found theoretically in the following manner:

Let the string be stretched between two fixed points O and A , Fig. 1, with a tension T . And suppose that it is uniform throughout, of radius r , density ρ , and length l .

Then, if disturbed slightly, and the motion be referred to three rectangular axes at O , of which OA is the axis of x , it

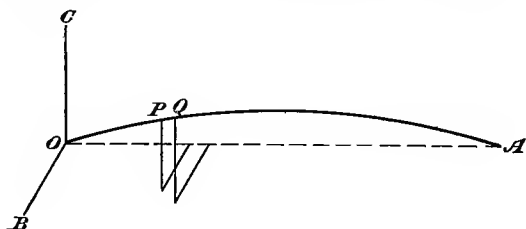


Fig. 1.

is obvious that any element such as PQ whose mass is ρds will have motions parallel to OB and to OC . The equations of motion of this element, on the hypothesis that the tension remains constant and that the length ds is equal to its projection dx , will be

$$\rho ds \frac{d^2 y}{dt^2} + T \frac{dy}{ds} - \left\{ T \frac{dy}{ds} + \frac{d}{ds} \left(T \frac{dy}{ds} \right) ds \right\} = 0,$$

$$\rho ds \frac{d^2 z}{dt^2} + T \frac{dz}{ds} - \left\{ T \frac{dz}{ds} + \frac{d}{ds} \left(T \frac{dz}{ds} \right) ds \right\} = 0,$$

the coördinates of the point P being x, y, z . For, since the tension at P along the tangent is T , the tensions along lines through P , parallel to OB and OC , will be $T \frac{dy}{ds}$, $T \frac{dz}{ds}$; and at Q , in the same directions, the tensions will be

$$T \frac{dy}{ds} + \frac{d}{ds} \left(T \frac{dy}{ds} \right) ds, \text{ and } T \frac{dz}{ds} + \frac{d}{ds} \left(T \frac{dz}{ds} \right) ds.$$

These become

$$\frac{d^2 y}{dt^2} = \frac{T}{\rho} \cdot \frac{d^2 y}{dx^2}$$

$$\frac{d^2 z}{dt^2} = \frac{T}{\rho} \cdot \frac{d^2 z}{dx^2}$$

since $ds = dx$; and they express analytically the fact that the string may move in the plane of xz , or in the plane of xy , or it may have both motions combined.

The solution of one gives

$$y = f(x - at) + F(x + at),$$

where $a^2 = \frac{T}{\rho}$.

This represents the transmission of arbitrary forms along the string with velocity a , where $a\tau = 2l$, τ being the periodic time of vibration of the fundamental note.

Hence
$$\tau = \frac{2l}{a} = 2rl \sqrt{\frac{\pi d}{Tg}},$$

where d is the specific gravity of the string referred to water as unity, and ρ therefore equal to $\frac{\pi r^2 d}{g}$, the cross-section of the string being circular. Therefore the pitch of the fundamental note of the string, which is equal to $\frac{1}{\tau}$, becomes

$$n = \frac{1}{2rl} \sqrt{\frac{Tg}{\pi d}}.$$

In this formula are included all the laws of the transverse vibrations of strings. It shews us that the pitch varies inversely as the length, and radius of the string, directly as the square root of the stretching force, and inversely as the square root of the density. All these facts may be verified experimentally by varying the elements of the string; and different experiments can easily be arranged to prove any particular law. The most useful and instructive exercise for the student, however, is to stretch a string over such a sonometer, as is shewn in Fig. 2.

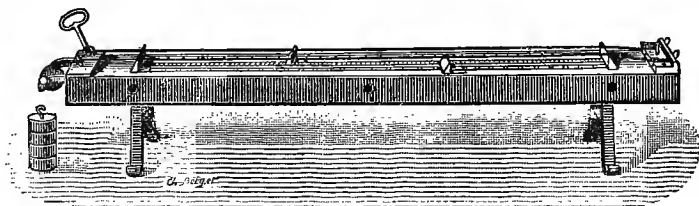


Fig. 2.

The string is fixed at one end, passes over two bridges which determine the two fixed points of the previous theory, and then, running over a pulley, is kept stretched by means of known weights. Its mean radius is determined with the wire gauge, its length is measured, or observed directly on the scale of the sonometer, and its specific gravity may be either taken from tables or found in the usual way. Then the pitch is calculated by the formula and verified by means of a standard fork.

This latter determination is sometimes difficult on account of the peculiar monotonous tone of the fork, which leads one to imagine that it is an octave lower than its real pitch, as well as the presence of loud overtones in the string (especially if made of steel) which an unmusical ear often chooses instead of the fundamental.

The easiest method of using the fork is to move the sliding bridge, usually provided with the sonometer, along until it cuts off a portion of the string which will vibrate in unison with the fork; then the pitch of the whole string will be to that of the

fork in the inverse ratio of the length of the open string to the part cut off by the bridge. The fork should be chosen of a suitable pitch so that the portion of the string cut off by the bridge may not be too small.

If the sonometer is provided with two sets of pins and pulleys, the experiment may be modified by stretching two strings of the same material, length, and diameter with different weights ; or by keeping the material, length, tension constant, and varying the diameters.

The most useful strings for experimental purposes are the fine steel *E* strings used for the mandolin and guitar. Copper or brass ones may also be used, as they can be obtained in all sizes, are comparatively uniform throughout, and require little tension to give a distinct note ; they are, however, if of copper, apt to stretch and break. Gut strings can be used, but they are hygroscopic, and it is difficult to get them uniform throughout their length.

IV. DETERMINATION OF PITCH.

(a) *Forks.*

The method employed for determining the pitch of a tuning fork is graphical. A slight style or writer of inappreciable weight is attached to the fork with wax ; the fork being so arranged that the style just rests against a cylinder turning at a known rate, and covered with a smoked paper. As the cylinder is turned the writer, owing to the vibrations of the fork, makes a wavy line on the paper, and the number of these vibrations made in a given time can then be counted, and the pitch of the fork found.

Generally, a standard fork is placed alongside of the fork whose pitch is to be found, and a comparison made between the two tracings on smoked paper ; thus rendering the determination independent of the rate at which the cylinder is turning if the points of the two styles are side by side. The style may be

made of a bristle or stiff hair, or of thin sheet brass ; but in the last case a small correction will have to be made unless the fork is very large.

(b) *Organ Pipes.*

The pitch of an organ pipe may be found most readily and simply by comparison with a fork, which may be chosen so as to give a small number of beats per second. Care must be taken, however, that the fundamental note of the pipe and not

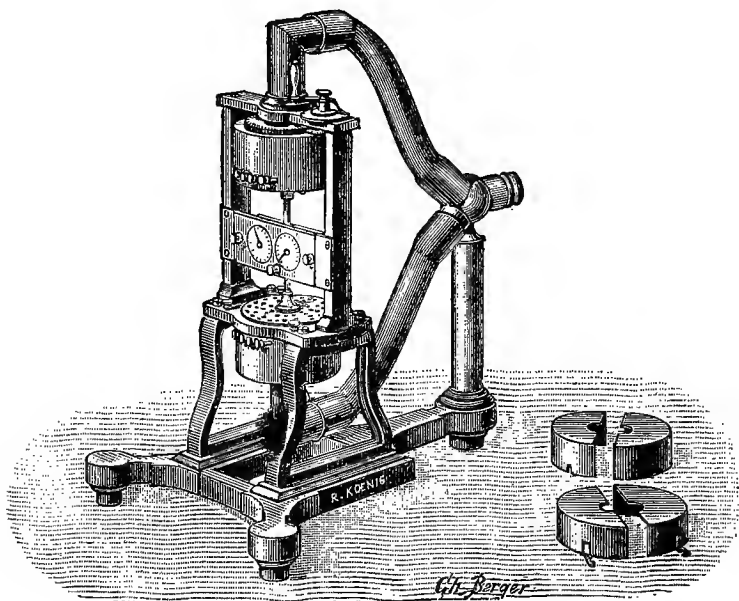


Fig. 3.

one of the overtones is chosen. Forks with sliding weights to alter their pitch are best adapted for this purpose.

One may also find the pitch of a pipe by means of *Helmholtz's siren*, which is shewn in Fig. 3.

The siren is attached to a bellows and driven at constant speed. Counters, shewn in the figure, are attached to the vertical axis of the instrument and enable one to observe the number of vibrations in any given time. To determine the

pitch of a pipe, the discs are driven until the note emitted by the siren is in unison with it, and if the note is maintained for some seconds without variation, the time and corresponding number of vibrations can be found. The method is tedious, and, unless great care is taken to maintain a constant pressure in the air chest of the bellows, no accurate result can be obtained. If the axis carrying the two discs is driven by a small electromotor, thus making the driving power independent of the note, better results may be obtained; the instrument is, however, much better suited for illustrative purposes on the lecture table, and is not strictly a measuring instrument. It may be mentioned here that the simplest form of siren, and one that may be used to some advantage for determining pitch, is a circular plate of brass with small openings symmetrically arranged. When driven uniformly by any rotation apparatus, and air, blown through a small orifice, is directed against the openings perpendicular to the face of the plate, a note is produced whose pitch can easily be found from the number of holes and the time of rotation. Such a plate provided with concentric rings of holes properly arranged may be used for lecture purposes to shew the formation of scales and combinations of notes, and has the advantage over all other forms of sirens on account of its perfect simplicity.

(c) *Strings.*

The pitch of a stretched string is found in the manner already explained, by comparison with a standard fork directly, or by calculation from the elements of the string where they are known.

When the fork chosen is not near the string in pitch, or where it is not possible to obtain a fork close enough, then the sliding bridge may be used on the string, and a portion cut off which vibrates in unison with the standard: the pitch of the the open string can then be inferred.

For *very low* or *very high* sounds, some difficulty is always experienced in determining pitch.

In the case of low sounds a siren may be used, but the difficulty then is to maintain a constant pressure in the air chest. The method devised by *Savart* may be used both for very low and very high notes. A wheel with a large number of teeth is driven uniformly at any desired speed, and a small strip of stiff cardboard or metal is held with an edge just touching the teeth; the noise, which is heard at first when the wheel rotates slowly, gradually develops into a decided musical note that becomes higher as the speed increases, the upper limit depending only on the rate at which it is possible to drive the wheel. When used to determine pitch, its note is made identical with the one to be observed, and the pitch calculated from the speed and the number of teeth. Conversely, the speed of a rapidly rotating wheel or disc may be best inferred from the note produced by some attachment that makes periodic sounds which change to a musical note under high speed. This method was used by *Wheatstone* in his measurement of the velocity of electricity, and is of some importance.

For very high notes, the sonometer with a sliding bridge, and a very tightly strung fine steel wire may also be used to advantage, and will give fairly accurate measurements of pitch as high as 10,000 vibrations per second.

Steel cylinders are also made, to furnish standards of pitch for very high notes, and to test the upper limit of audibility. They range from 8192 vibrations to 40,000, and are usually made to give the notes of the major diatonic scale. By means of them one may locate any particular high note within a few hundred vibrations, by the unassisted ear.

V. LISSAJOUS' OPTICAL METHOD OF TUNING.

(1) *With mirrors attached to the forks.*

Instead of tuning by means of the ear, another method, devised by *Lissajous*, is used for the accurate comparison of forks which may be either close to one another in pitch, or very

far apart. Mirrors are fixed rigidly to the prongs of the forks, which are so arranged that they can vibrate in two planes at right angles to one another; and a beam of light from a small luminous source is allowed to fall in succession on the two mirrors, and then received on a screen, viewed directly by the eye, or allowed to pass into a small telescope focussed so that the image of the luminous source is clear and sharp. On vibrating the forks, the image in the telescope is seen to pass through various forms owing to the compound motion; and, by loading the forks until certain known forms are obtained, the ratio of the two vibration numbers can be inferred.

Theory.—When the two forks vibrate, their motions are periodic, and the displacement of the spot of light seen after reflection from the two mirrors will therefore be equivalent to two displacements obtained by solving the equations of motion:

$$\left. \begin{aligned} \frac{d^2x}{dt^2} &= -\mu x \\ \frac{d^2y}{dt^2} &= -\mu' y \end{aligned} \right\}$$

These give

$$\left. \begin{aligned} x &= a \cos 2\pi \frac{t}{T} \\ y &= b \cos 2\pi \left(\frac{t}{T'} + \delta \right) \end{aligned} \right\}$$

Where a and b are the amplitudes of vibration, T and T' the periodic times, and δ a constant, called the *phase*, which depends on the initial circumstances of motion. On eliminating t , we shall get an equation to a curve, which is the form seen in the telescope.

1. If $T = T'$, the forks will be in unison, and the curve seen will have the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos 2\pi\delta = \sin^2 2\pi\delta,$$

which represents an ellipse or straight line, as shewn in (a) Fig. 4, according to the values of δ , determined by the initial conditions of vibration.

If T differs slightly from T' , then, whatever be the initial form, the curve begins to change, and runs through all the forms shewn; on loading the forks properly with small pellets of wax the change takes place more slowly, until when T becomes equal to T' the figure initially obtained preserves its form throughout the motion.

2. If $T' = 2T$, then, on eliminating t as before, a complicated equation is obtained, which represents the curves shewn in (b).

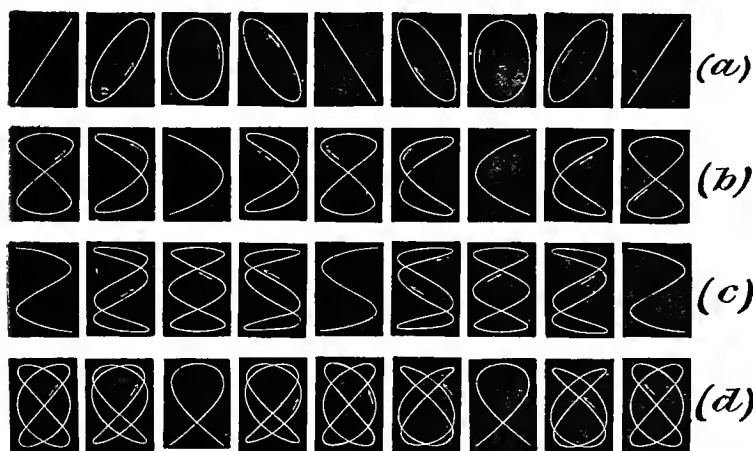


Fig. 4.

In a particular case, where $\delta = \frac{1}{4}$, the equation reduces to a parabola.

3. The figures in (c) and (d) shew the forms obtained when $T' = 3T$, and $2T' = 3T$.

There are many ways of drawing these curves geometrically; and an excellent exercise for the student is to take a rectangle whose sides represent the amplitudes and map out a series of points from the relations for the displacements, for any chosen

ratio, assuming some particular value for δ . The curve for that particular phase is then easily drawn.

(2) *The vibration microscope.*

For the more practical purpose of comparing forks with standards, an instrument known as the *vibration microscope* is used. It is represented in Fig. 5. A standard fork of known pitch, which can usually be altered by sliding weights, carries

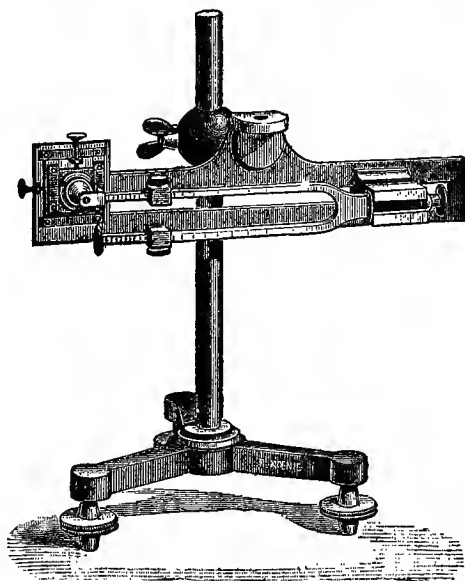


Fig. 5.

on one prong a small lens, whose axis is perpendicular to the plane of vibration of the fork; and a fixed microscope is arranged independently behind, so that its axis can be placed in the prolongation of the axis of the lens. The lens on the fork forms the objective of the microscope. The fork to be examined is then placed so that its plane of vibration is perpendicular to that of the standard; and any small mark on it is viewed through the microscope, which thus shews an image

of the mark going through a compound motion, when the two forks are vibrating, similar to that obtained with two mirrors and a spot of light. The pitch of the fork is inferred as in the former experiment. Practically, this furnishes an exact way of reproducing a standard; by filing slightly the ends of the prongs the pitch can be raised as much as one pleases, and if it becomes too high a slight filing on the inside of the prongs lowers it; and by operating in this way until the stationary ellipse or straight line is obtained, a standard may be made in a short time. The accuracy of the method is such that one may get two forks so close in pitch that they will not differ from one another by more than one beat in a minute, a thing which no system of tuning by ear could ever accomplish.

VI. HARMONIC MOTION.

(1) *Composition of parallel and rectangular vibrations.*

A motion represented by the equation

$$\frac{d^2x}{dt^2} = -\mu x.$$

is said to be *periodic* or *harmonic*. The prong of a tuning fork gives such a motion, as may be seen graphically by arranging a small writer on the end of one prong of the fork; when put in vibration and drawn so that the writer moves uniformly along a piece of smoked glass, the tracing is an accurate curve of sines.

Two parallel or rectangular harmonic motions may be compounded, by aid of two tuning forks, with the apparatus shewn in Fig. 6.

One fork is fixed in position, and holds on one prong a plate of smoked glass which vibrates with it; on the other prong is a small sliding weight which is used as a counterpoise to the glass plate. The other fork, provided with a small writer, is arranged independently so that it can be drawn backwards while the

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writer rests gently on the smoked glass. To compound two parallel motions, the forks are so placed that their prongs and the plate of glass are all parallel; the position for angular com-

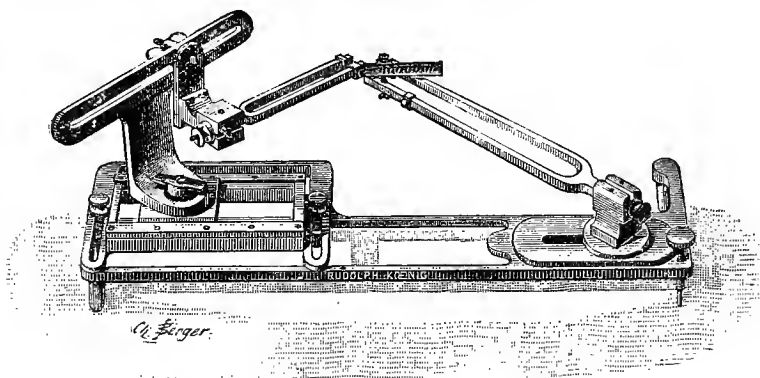


Fig. 6.

position is shewn in the figure. Both forks are set vibrating, and then the one carrying the writer is drawn along as uni-

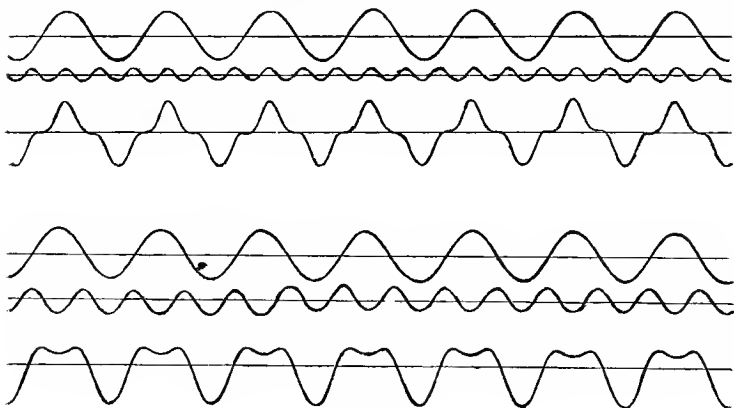


Fig. 7.

formly as possible, and the resulting curve traced on the glass shews the composition.

These curves may also be drawn geometrically, and, as in

the case of Lissajous' curves, the drawing is a good training for the proper appreciation of wave motion.

Figure 7 shews two compositions of parallel motions. The lower represents two curves of sines of which one wave length is twice the other, and their composition is also shewn. The upper one shews the composition of two curves whose wave lengths are as 1 to 3.

Figure 8 shews the composition of two curves described by forks whose vibration numbers are in the ratio of 4 to 5.

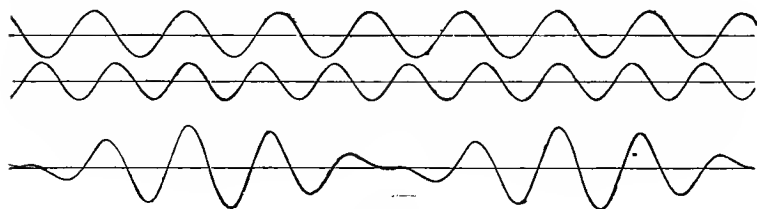


Fig. 8.

Figures 9, 10, 11 shew compositions arising from forks of 1 : 2, 1 : 3, 1 : 4 very nearly. One is exactly in the ratio 1 : 4. The curious chainlike appearance presented by them is caused by the slight difference of phase.

Figure 12 shews the composition of two vibrations nearly rectangular.

Figure 13 represents to the eye the effect produced on the ear by two sets of waves which have very nearly the same periodic time. It is, in fact, a graphical representation of a *beat*.

(2) *Blackburn's pendulum.*

The composition of two motions at right angles to one another may also be exhibited by means of a very simple contrivance known as *Blackburn's pendulum*. It is shewn in Fig. 14.

Three wires are joined together at *D*, the two upper ones being fastened to a horizontal beam, while the lower carries a heavy sphere to which a small writer is attached. If the sphere be moved perpendicularly to the plane of the paper, it oscillates about the point *E*, its time of oscillation varying as the square

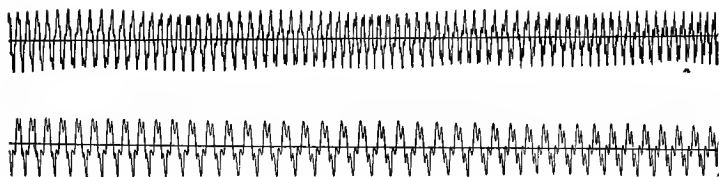


Fig 9.



Fig. 10.

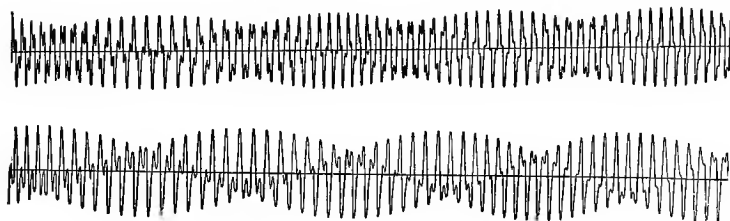


Fig. 11.

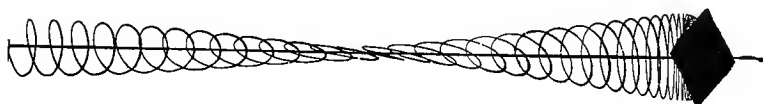


Fig. 12.



Fig. 13.

root of EC , C being the center of the sphere. If disturbed in the plane of the paper, it oscillates about D , with a different periodic time. The lengths, EC , DC , can be adjusted so that the times of oscillation are in any desired ratio. If then the sphere be moved indifferently in any direction, and a piece of smoked glass be placed underneath so that the writer is always in contact with it, a closed curve will be traced out, arising from the compositions of the two rectangular motions.

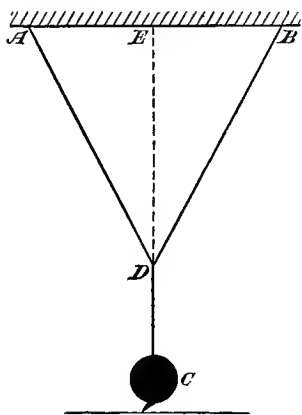


Fig. 14.

suspended by steel wires. The writer was a piece of thin sheet brass.

Figure 16 is a photograph of an actual tracing made with a rough pendulum consisting of an iron ball

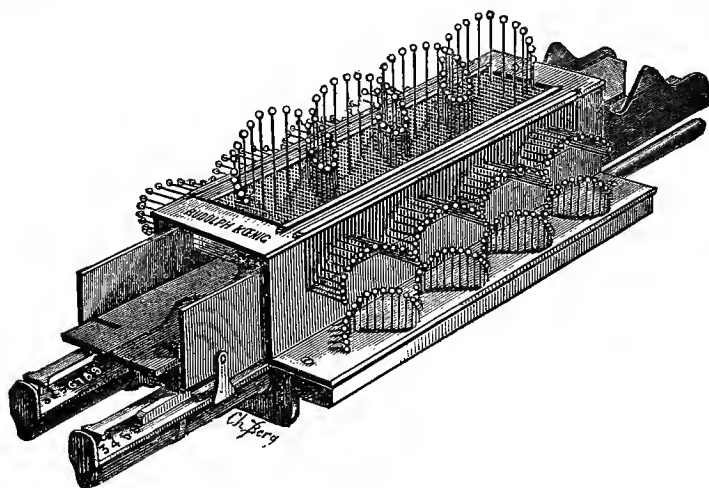
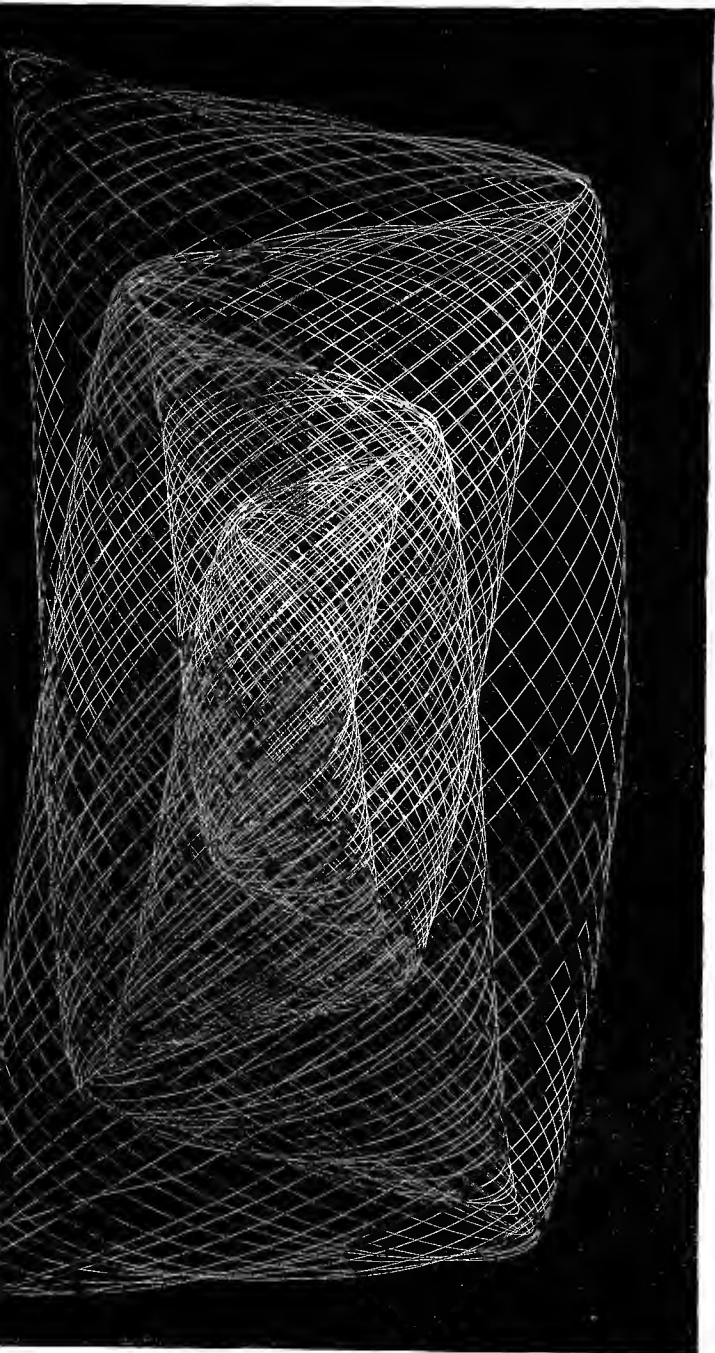


Fig. 15.

(3) *Wheatstone's wave apparatus.*

This shewn in Fig. 15 is an admirable way of studying the



composition of parallel and rectangular vibrations; also for illustrating the general phenomena of wave motion.

The machine has a number of stiff iron rods, each made in the form of a cross and carrying beads at the ends of the upper and the two side pieces. These rods are constrained to move in one plane. Boards are supplied with the apparatus, which have curves of sines cut along their edges, and are made in pairs so that the curves fit one another. The iron rods are so arranged that when the boards are placed in position and pushed through the machine, the motion of the beads shews the propagation of the wave form, which may be either simple or complex. If the horizontal boards are used alone, the beads have rectilinear motions, which indicate the propagation of a plane horizontal wave; if the vertical boards are used, plane vertical waves, resembling somewhat water waves, are obtained; and, by combining vertical with horizontal, compound waves like those of light may be formed.

It will be noticed in all cases that, whatever boards or combinations be used, the curve on top is the resultant of the two curves shewn at either side by the sets of fixed and movable beads.

VII. OVERTONES.

(1) In *strings vibrating freely*, the complex nature of the vibration, as determined analytically from the equations of motion by the aid of Fourier's theorem, may be seen directly by the eye, or observed by the ear with the assistance of what are called *resonators*; these are hollow globes or cylinders of metal or glass, constructed of various sizes so that the air inside vibrates in sympathy with certain corresponding notes, and thereby reinforcing them renders it possible for a person to hear tones which, without the resonators, would not be audible. They are usually made in a series corresponding to the notes of the *harmonic scale*, and may be either fixed or variable. Figure 17 represents two kinds, one fixed and spherical

in shape, the other cylindrical and capable of adjustment so as to be used for several tones.

To use them properly, the smaller end is either placed directly in the aperture of the ear or connected with it by means of a short rubber tube; the other end being then brought into the vicinity of the complex note to be examined. Success in the experiment is best attained by closing, at the same time, the aperture of the other ear. To obtain good results from a string, choose a steel one about six feet long and stretch it between two fixed supports until it gives a pitch of 256 v.s. With the series of resonators corresponding to this pitch as fundamental, it is possible to recognize eighteen or twenty pure overtones of the harmonic scale. In fact, after



Fig. 17.

a little practice with the resonators, they may be discarded and most of the overtones still heard by the unassisted ear. With steel and copper strings, the third harmonic is often louder than the fundamental; and it is, no doubt, owing to the presence of these overtones in the string that great difficulty is always experienced in tuning a string to a fork, as the ear is apt to choose, instead of the fundamental note, one of the more prominent overtones.

(2) In the case of *organ pipes*, the complex nature of the note obtained may also be shewn by using the resonators; but the best method for the examination of the notes produced by both closed and open pipes is that of the manometric flame, in connection with the resonator. See Exps. 12 and 13.

(3) The *tuning fork* is, in reality, a combination of two steel rods which vibrate in unison with one another; and its pitch varies directly as the thickness of the prongs (measured in the plane of vibration), inversely as the square of the length, and is practically independent of the thickness of the prongs (measured perpendicularly to the plane of vibration). The prongs in vibrating, may divide into segments, just like a string; and therefore may be made to give tones superior to what is commonly called the pitch of the fork. These tones, however, are not true harmonics: the first one being nearly six times, and the next



Fig. 18.

higher, seventeen times, the pitch of the fork. To attain the first, the fork is bowed at the side, about half way down; the second may be obtained by bowing at a point one third of the way from the bottom. Or they may be obtained along with the fundamental by bowing the fork first in the usual way on top and then along the sides. Figure 18 shews tracings of a fork accompanied by its superior tones.

The lower right-hand tracing is made by the fundamental with its first superior tone; the lower left hand one is the fundamental with the second superior tone; and the upper one is a combination of the fundamental with the first two superior tones.

VIII. THE CHRONOGRAPH.

This is an instrument which is used for the measurement of small intervals of time by means of an electric interrupter and tuning fork. The arrangement is shewn in Fig. 19.

A tuning fork, held upright in a rigid support, can be put in vibration by a current of electricity, which passes along one prong of the fork, through a small contact piece that acts as an

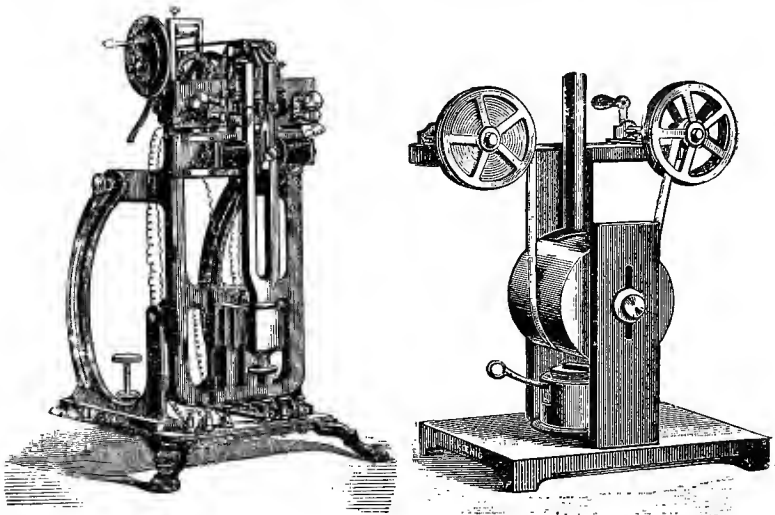


Fig. 19.

interrupter, and then through two electro magnets situated one on each side of the fork, as shewn in the figure. To the end of one of the prongs of this fork is attached a writer of brass which rests against a piece of smoked paper, so arranged in a roll that it can be drawn along continuously by the handle shewn at the upper part of the apparatus. Two small writers, one on each side of the tuning-fork writer, act as interrupters, when the currents passing through the electro-magnets which carry the writers are in any way broken. When the fork is running elec-

trically, it makes a wavy line on the smoked paper as it is unrolled, each distance from crest to crest corresponding to $\frac{1}{n}$ seconds, where n is the pitch of the fork in double vibrations. The two interrupters at the side make continuous straight lines. If now the interrupters are arranged in a circuit which can be broken so as to indicate the beginning and the end of any particular phenomenon, then, if the interruptions are properly made while the paper is moving and the fork vibrating, marks will be visible on the smoked paper where the tracers of the interrupters have departed from a straight line. By counting the number of waves between these marks the time of the event is found. Instead of using two interrupters, one only may be used; although in using both, one serves as a check upon the other. The two interrupters can also be arranged in separate circuits independent of one another, while the tuning-fork circuit is, of course, independent of both.

With such an instrument as this, and with due care, one may measure with great accuracy the time of vibration of a body, or the time of a falling body, or any such interval where it is possible to form electrical circuits with the interrupters.

The auxiliary figure represents an apparatus for smoking the roll of paper in a proper manner. A hollow brass cylinder is fixed in position between two wooden supports, and the paper is unrolled from one grooved wheel to another, passing around the cylinder as shewn in the figure and being blackened by a coal oil lamp which is placed underneath the cylinder. To avoid burning the paper the cylinder is partially filled with water.

IX. THE CLOCK FORK.

This (Fig. 20) consists of a fork of 100 v.d., held upright in a firm support and connected to a clockwork movement in such a manner that it regulates it through the escapement just as a pendulum would; but it receives at each oscillation a small impulse which enables it to keep up a steady vibration.

The two branches of the fork carry sliding weights attached to micrometer screws so that the period of vibration can be regulated with great precision. In addition, one of the prongs carries the objective of a microscope whose ocular is placed on an independent support; thus, it may be used in the same man-

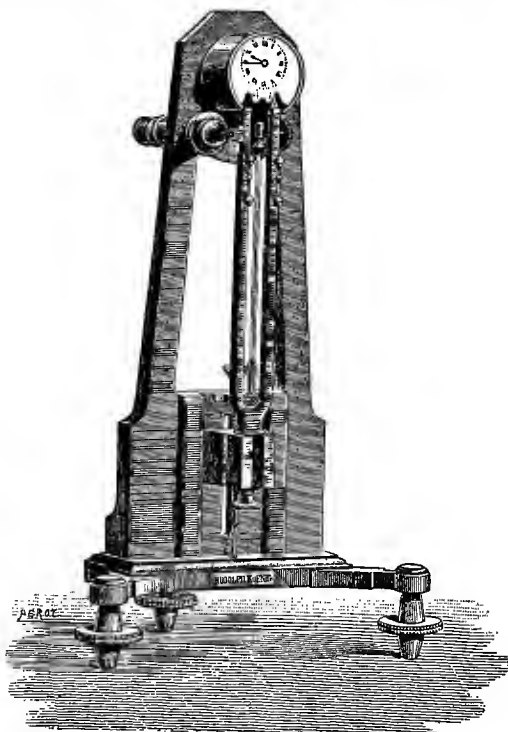


Fig. 20.

ner as the *optical comparator* of *Lissajous*. A thermometer placed between the prongs of the fork gives the temperature.

The instrument gives perfect results when properly regulated, and enables one to observe the effects of temperature on the period of vibration of a tuning fork by using an auxiliary fork whose temperature is gradually increased, and noticing the

figures obtained with the aid of the vibration microscope. It serves also to study in a similar way the effect of *reinforcement* on the pitch of a fork by observing the vibration of a fork placed on supports of different materials and forms. It may be stated here that the ordinary steel forks between 128 v.s. and 1024 v.s. have their pitch diminished by about $\frac{1}{9000}$ for each increase of temperature of 1° C. This coefficient increases slightly as the pitch of the fork increases, and also changes considerably with the material of the fork.

X. MELDE'S EXPERIMENTS.

A tuning fork has a string attached to one prong and stretched over a pulley by a small weight, or else depending vertically from the fork. When put in vibration, the fork sends waves along the string which will be transversal or longitudinal according to the position of the string relatively to the plane of vibration of the fork. The string then divides up into loops and nodal points according to the well-known laws of stretched strings explained in Exp. 3. A simple experiment for transverse vibrations is to alter the length and tension, and prove that the number of loops obtained varies inversely as the square root of the tension. The fork may be kept vibrating electrically by using an ordinary dry or mercurial interrupter. Strings of various kinds may be used to prove the law of density: silk cord, fine steel, copper, or platinum wires give good results.

The complex vibration of a string attached to two forks may be shewn by means of the apparatus indicated in Fig. 21, where the forks are vibrated electrically and the string is stretched between them.

Curious and instructive complex motions are obtained by using a silk cord about a millimeter in diameter and a meter in length stretched from a fork (vibrating electrically) of 128 v.s. On placing the string so that it makes an angle of 45° with the fork in its plane of vibration, and stretching it with the proper

weight (which can easily be found by trial or by calculation), one sees it divide up into two portions which resemble solid figures whose cross-sections are usually parabolas or lemniscates, and in the center is a plane.

This seems to arise from the two sets of waves which are travelling along the string with velocities in the ratio 1 : 2.

Tyndall's experiment with the luminous wire may also be shewn by using an electrically vibrating fork and a platinum wire a millimeter in diameter and a meter long, which runs over a brass pulley and is stretched by the proper weight. An independent current is then passed along the wire by one prong

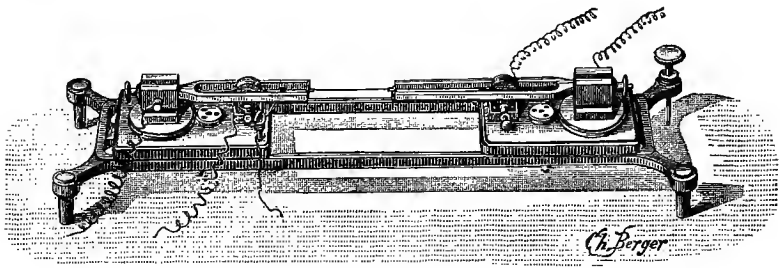


Fig. 21.

and the brass pulley which gives good contact ; the wire being then made red hot by a current of 10 or 15 amperes, the fork is set vibrating ; when immediately one sees the loops become dark, owing to cooling by their rapid motion, and the nodes grow brighter. More current is then turned on until the loops again become bright, and the whole outline is seen ; the wire is then very apt to break at the pulley, or at a nodal point, owing to the unequal distribution of heat. A heavy steel wire may also be used for this purpose, but in that case the weights must be adjusted after it becomes red hot, on account of its losing its elasticity. No doubt the best results would be obtained by using a carbon filament, such as is seen in an incandescent lamp, since it possesses the advantage of having its resistance diminished as the temperature is increased.

XI. HELMHOLTZ'S APPARATUS FOR SYNTHESIS.

This is shewn in the adjoining figure. A series of forks running from ut_2 to mi_8 are arranged in the same electrical circuit with an interrupting fork. Resonators are placed behind each fork, and are capable of small adjustments so that they may be moved backwards and forwards.

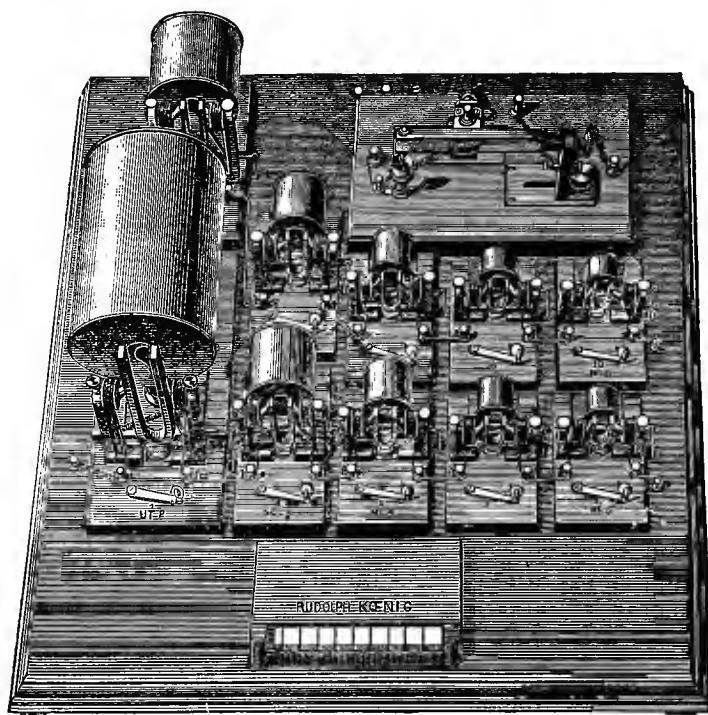


Fig. 22.

Stoppers are provided for the resonators and connected by means of strings with a keyboard in front, in such a way that on depressing any key the corresponding fork is heard vibrating loudly. When the keys are all down and the stoppers closed, all that is heard is a gentle humming sound.

The object of the apparatus is to combine the fundamental note ut_2 with any of the harmonics in the series, and observe the quality or *timbre* of the corresponding tone. It does not admit of accurate comparison, but yet affords an instructive exercise for the student. The noise of the interrupting fork may be overcome to a large extent by dividing the current, or

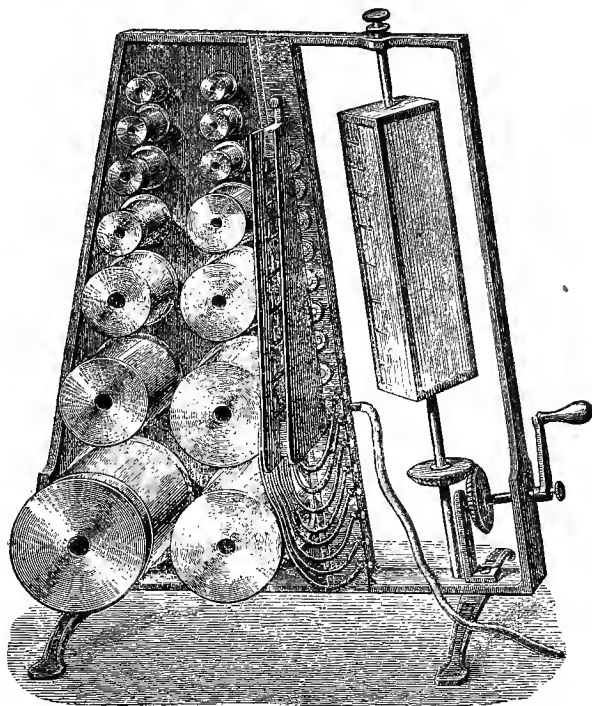


Fig. 23.

introducing a great resistance into the circuit; or the interrupting apparatus, which is quite independent of the other part of the instrument, may be carried to another room, and the circuit through the forks completed by wires leading from one room to the other.

XII. KÖNIG'S ANALYZER.

This is the reverse of the apparatus described in the preceding experiment: it is used to analyze the constituents of a compound note. The general appearance of the instrument is shewn in Fig. 23.

The principle used is that of the *manometric flame*, devised by *König*, combined with the rotating mirror of *Wheatstone*. In Fig. 24 a section of the *manometric capsule* is given. A small wooden box encloses a membrane *M*, which is stretched between two points *B* and *C* and separates an air chamber from a gas chamber. The gas is supplied at *G*, and burns at *F* with a small narrow flame about half an inch in height. The air from any source enters at *A*.

In the analyzing apparatus a series of these manometric capsules have their air chambers connected with the resonators

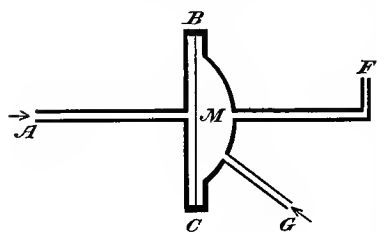


Fig. 24.

by tubes at the back (not shewn in the figure). The tubes seen in front lead from the gas chambers to the large tube at the bottom which supplies gas to them all. Stop-cocks are provided for each gas jet, so that any one may be shut off if necessary.

In order to examine any compound note, such as that produced by an open pipe, its harmonics are calculated from the pitch of its fundamental, the resonators are adjusted for these harmonics, and when the pipe is blown and held near the front of the apparatus, while the mirror (shewn at the right) is turning, certain of the flames will be seen to give wavy appearances in the mirror, shewing that they are vibrating, by the intermediary of the elastic membranes, in sympathy with the air in certain of the resonators.

The instrument furnishes a useful means of observing over-

tones from any source of sound ; but it must be adjusted with considerable care in order to perform its work with accuracy.

XIII. MANOMETRIC FLAMES.

The principle of the manometric flame may also be used for many other purposes : notably for observing the complexity of the vowel sounds or of notes sung by the human voice, for

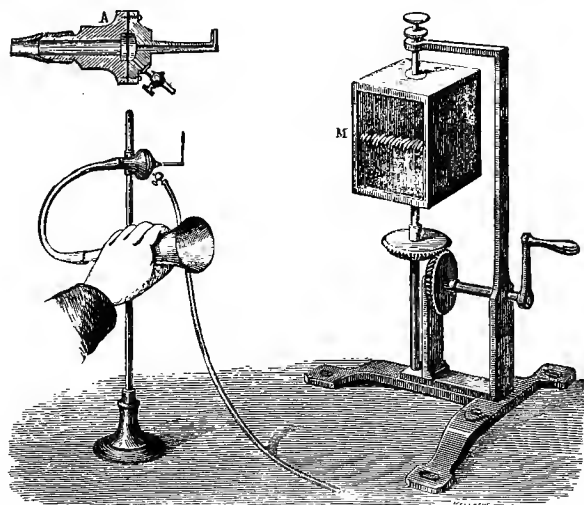


Fig. 25.

showing interference of waves of sound, and for the general composition of wave motions.

Figure 25 shews how it may be used for the examination of the vowels sung according to any chosen pitch.

The manometric capsule, a section of which is at *A*, is attached by means of a tube to a funnel-shaped mouthpiece for collecting the waves of sound. The rotating mirror is shewn at *M*.

Figure 26 explains the manner of comparing the vibrations of two columns of air in organ pipes ; and, by connecting the two

gas tubes leading from the capsules in the organ pipes with the two ends of a T-shaped tube, a single gas jet can be obtained which is actuated by the two sets of vibrations simultaneously.

A number of these combinational vibrations are represented

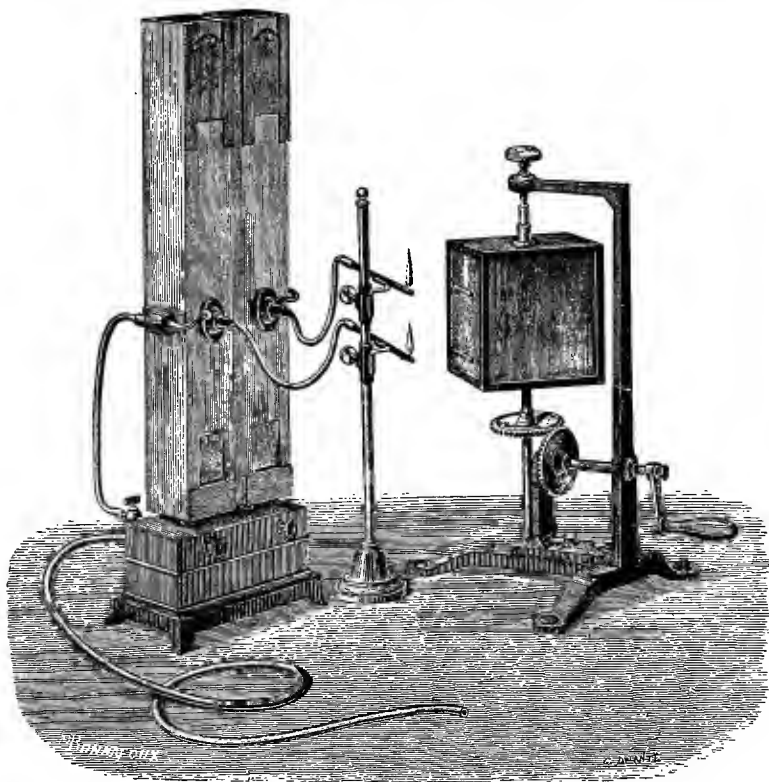


Fig. 26.

below: they have been obtained in the manner indicated, and correspond to the intervals of the major diatonic scale.

The manometric flames may also be employed in conjunction with the resonators of tuning forks. In most of the resonators supplied by König with his forks, there is a small brass tube at the back provided with a stopper, the use of which is to con-

nect the resonator with a manometric flame by means of a long rubber tube and thus observe the vibrations of the air within. Any number of forks may thus be used and the composition

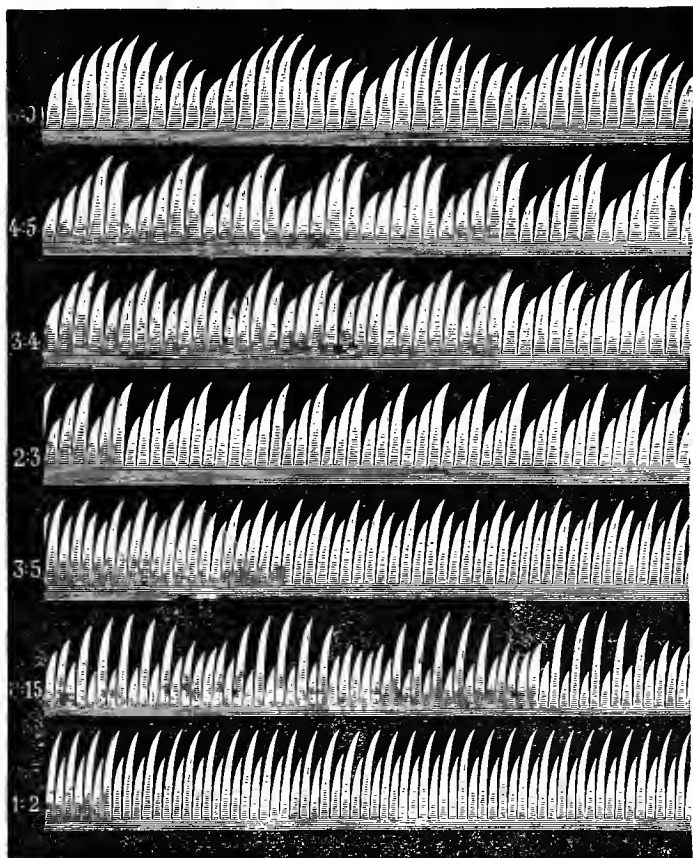


Fig. 27.

of their vibrations studied by connecting their resonators all to a common gas jet.

If forks forming a harmonic scale be chosen, one may com-

bine them optically and *see* the effect which is obtained by the ear with the apparatus of Helmholtz.

Perhaps the most striking experiment is that of König for

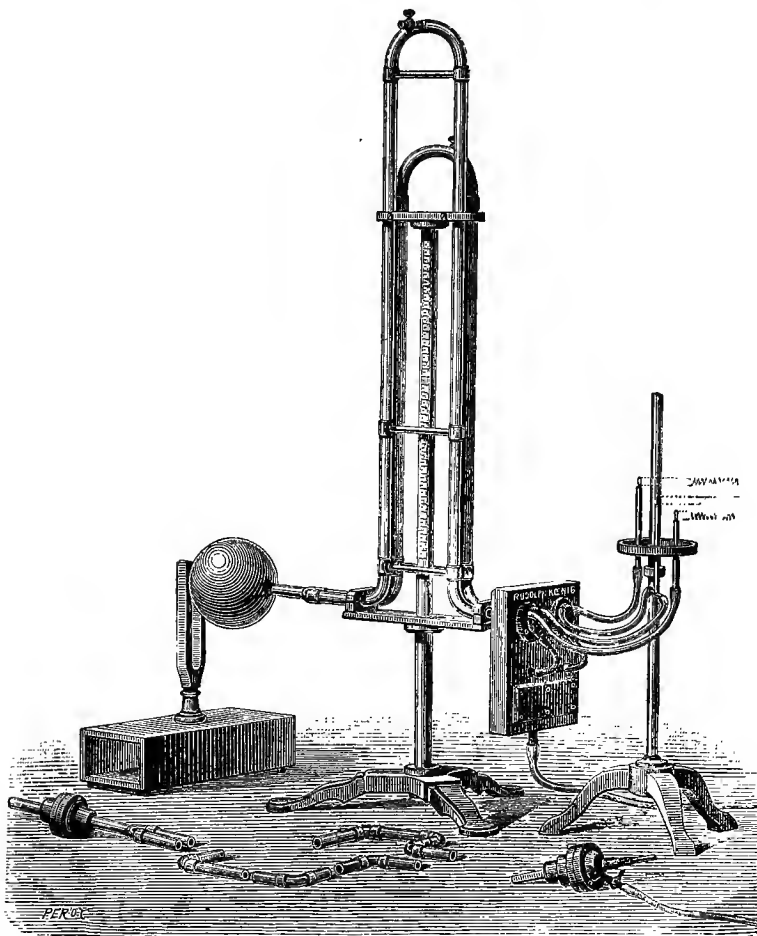


Fig. 28.

showing interference. The apparatus shewn above is constructed in accordance with the principle introduced by Herschel.

A tuning fork sends vibrations, reinforced by a resonator, to a tube which, between its extremities, is divided into two branches, the length of one of which can, by being drawn out, be altered at will. The vibrations are sent to two independent gas jets, after having travelled different distances, and a third gas jet enables one to see the effect of compounding the two sets of waves. The manometric capsules are inserted between the gas jets and the tubes.

The instrument admits of perfect adjustment, and when complete interference of the two wave motions takes place the appearance of the three flames, as seen in a rotating mirror, is as shewn in Fig. 29.

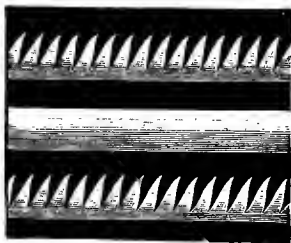


Fig. 29.

XIV. VELOCITY OF SOUND IN AIR.

The velocity with which sound is propagated through any medium will evidently depend on the elasticity and density of this medium, and theoretically it may be calculated from the formula

$$V = \sqrt{\frac{E}{D}},$$

where E is the coefficient of elasticity and D the density.

In *solids* E and D can be found experimentally, and the determination of the velocity presents no difficulty.

For *liquids* the above formula becomes

$$V = \sqrt{\frac{g\rho H}{KD}},$$

where ρ is the density of mercury, H the height of the normal barometer, K the coefficient of compressibility, and D the density of the liquid at the observed temperature. But in the case

of air or any *gas* an experimental method of finding V directly is preferable on account of the difficulty of determining accurately the heat constants which are involved in E .

A method that at first sight appears simple is the direct one of firing a gun at a known distance from an observer, who estimates in some way the time which elapses between the perception of light and that of the sound; but the personal errors in the appreciation of this interval seriously interfere with the accuracy of the result, and the method at best is but approximate.

Other methods are given here to illustrate the various ways in which the velocity of sound in air may be measured; modifications in some of them would, no doubt, insure greater accuracy.

(1) *By measurement of a wave length.*

We know that when a periodic disturbance, for example that from a vibrating fork, is propagated through the air, any particle of this air is in the same state of vibration at successive times separated by an interval T which is therefore called the *periodic time* and is equal to $\frac{1}{N}$, where N is the pitch in double vibrations; and the *wave length* is defined to be the distance which the wave form travels in this time T .

Hence we have

$$\lambda = VT,$$

if V be the velocity of propagation and λ the wave length. If then we take a tuning fork and measure λ in any way, we can calculate V .

With forks of very low pitch and consequently of great wave length this can be done with some degree of exactness by aid of the ear, either unassisted or with a resonator. The fork is put in vibration in front of a reflecting surface, and the observer chooses points in the surrounding medium where maximum or minimum effects are produced on the ear. A measurement of the distance between successive maxima or minima gives half

the wave length ; and the pitch being generally given accurately on the fork, we have

$$V = \lambda N.$$

An improvement on this method might be made by using resonators in connection with manometric flames.

The most accurate way perhaps to estimate the wave length of a fork is to use the apparatus shewn in Fig. 28, and, after obtaining complete interference, to measure the wave length by means of the scale attached to the apparatus.

Assuming the pitch of the fork, the velocity of sound can then be inferred. The same apparatus may also be used for comparing the velocities of sound in different gases.

(2) *By resonance.*

If a hollow cylinder, open at one end, be fitted at the other with a tight plug and piston rod which can be moved backwards or forwards, and a tuning fork be put in vibration and held with the prongs over the open end, it will be found that, by adjusting the movable plug, a point is reached where the air column within the cylinder vibrates in sympathy with the fork, and acts as a *resonator*. In this case, since $\lambda = VT$,

$$V = 4 l N$$

where l is the length of the vibrating air column and N the pitch of the fork in double vibrations.

The method is not exact, a considerable error arising from the size of the open end of the cylinder. *Rayleigh's* correction for this gives

$$V = 4 (l + r) N$$

where r is the radius of the tube supposed cylindrical. If the tube be of sufficient length, it is evident that a series of maximum reinforcements may be found, and then

$$V = \frac{4 (l + r) N}{2n + 1}$$

where n is zero or any integer.

The adjoining figure shews how the experiment may be tried. A tube AB a few feet in length is attached by means of a clamp at B to another tube with a reservoir R , and is provided with two stopcocks t, t , for the regulation of the supply of water from the reservoir. Adjustment is then made until the distance AL gives perfect resonance.

(3) *Bosscha's method.*

This method, although not more exact than the preceding, involves a new idea, that of noting by ear the coincidence of

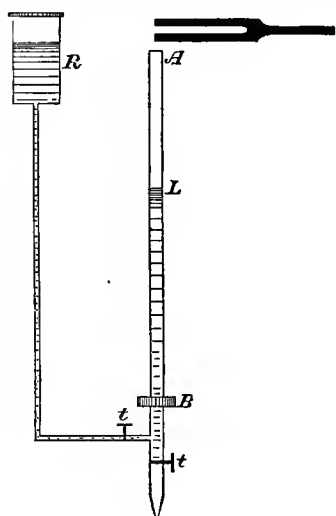


Fig. 30.

two sounds. The apparatus, devised by König, consists of two boxes provided with sounders and electro-magnets, which are inserted in an electrical circuit with a mercurial interrupter, whose period can be regulated by means of adjustable masses. The axis of the interrupter carries a mirror, and a fixed tuning fork of 40 v.d. also carrying a mirror is so situated that the image of a silvered bead is seen by reflection from the two mirrors. The fork and interrupter vibrate in perpendicular planes; and the time of interruption is

found from the figure given by the image of the bead. It is usually made one tenth, or one eighth of a second, the corresponding figures having then four or five loops. When the adjustment to get the correct time of interruption has been made the sounders are set in motion, and of course give simultaneous raps when heard by an ear near them; one of them is then carried away by the observer, who then hears the raps coming alternately and then together again. He notes the distance at which they appear coincident: and this

distance, divided by the period of interruption, must give the velocity of sound.

If then V = velocity of sound in air at zero,

t = mean temperature of the air,

T = period of interruption,

d = distance of coincidence,

then
$$V = \frac{d}{T\sqrt{1+\alpha t}},$$

where $\alpha = .003665$, and t is the mean temperature of the air. By taking two or three coincidences and varying the period of interruption fairly good results may be obtained; but the error in estimating a single coincidence by the ear, however skilled it may be, may amount to as much as one foot: so that the velocity is liable to be in error eight or ten feet per second.

(4) *Kundt's method.*

This enables one to compare velocities in gases with one another; and also the velocity in air or any gas with that in a solid. It is susceptible of great accuracy and the apparatus is simple and easily arranged.

A glass tube AB rests freely on two supports, and at one end is a movable plunger C , and at the other end a cork B , through

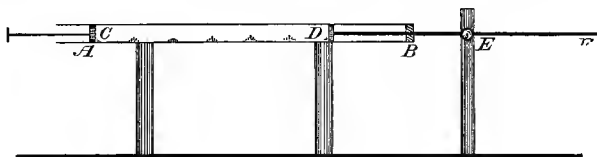


Fig. 31.

which passes a metal rod DF , fixed at its center E , and having at D a small circular disc which fits the tube loosely. If DF be put in longitudinal vibration and the length CD properly adjusted by the plunger, it will be found that nodes and loops are formed in the air inside; and if lycopodium be strewn lightly

within the tube it will collect and form little heaps where the vibrating air is at rest ; and it is evident that the rate of propagation of sound in air is to that in the metal DF as the distance between two loops (or which is the same thing, two nodes) is to

the length of the rod. If then v is the velocity of sound in air, V that in the metal, l the distance between the little loops of powder, L the length of the rod,

$$v = V \frac{l}{L}.$$

By placing a similar tube in connection with the other end F , and using another gas in it, the velocities of sound in air and in the other gas will evidently be to one another as the distances between the respective nodal points.

(5) *Regnault's method.*

By far the most accurate method is that devised by Regnault, who determined by direct measurement of distance and time the velocity of propagation of

sound in tubes of various diameters ; and the measurement of time was made independent of the observer, thus eliminating the personal error. A modified form of the apparatus used by him, which enables one to perform the experiment in a limited space, is shewn in Fig. 32.



Fig. 32.

A series of tubes is arranged, forming one long tube, and an electro-magnetic arrangement at one end is attached to a pistol which, when discharged, produces a disturbance that travels to the other end and there causes a mark to be made on some recording apparatus, or can, if necessary, be reflected, return and again travel to the other end, and so on. The pistol is also connected with the recording apparatus, which may be either a cylinder with tuning-fork attachment or the *chronograph* of Exp. 8.

The distance along the axis of the tube can easily be measured: in the apparatus constructed by König it is about 30 meters.

Besides being used for measuring the velocity of sound, this instrument enables one to investigate all the phenomena of reflected sound waves by attaching at the reflecting end tubes of various lengths, which are put in communication with manometric flames.

XV. DÖPPLER'S PRINCIPLE.

When a source of sound has a motion of translation an observer who listens will hear the sound gradually changing in pitch. This may very often be noticed in the case of a railway locomotive, which whistles as it is coming into a station: to an observer at the station the pitch of the note seems gradually to rise.

If V be the velocity of propagation, v the velocity of translation, and λ , λ' the wave lengths of the two notes, then

$$\frac{\lambda'}{\lambda} = \frac{V \pm v}{V}$$

according as v is in the same or opposite direction. One of the simplest ways to study this is by means of two tuning forks of rather high pitch (512 and 520 v.s.), which give four beats per second when sounded together; if an observer stations himself

at a distance of thirty or forty feet, and an assistant carries one fork towards him, it will be found that the number of beats alters; if the higher fork be carried at the rate of about two feet per second, then nearly five beats a second will be heard; if the lower fork, then about three.

This method might be made accurate by having the tuning fork slide along, at a uniform rate, a board twenty or thirty feet long, by means of a pulley and weights, which could be adjusted to give the proper speed.

The experiment can also be easily tried by rotating a tuning fork, or by placing it on the slide of a slowly moving horizontal engine.



Fig. 33.

The apparatus of *Mach*, to illustrate this principle, consists of a hollow brass tube with reed attachment at one end or at both ends. This can be rotated while the reed is vibrated through a side tube, as shewn in Fig. 33, and an observer stationed in front hears only one note, while in the plane of rotation he hears the alternate increase and diminution in pitch. The instrument in this state, however, hardly admits of giving measurements.

HEAT.



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EXPERIMENTS.

I. DETERMINATION OF THE ZERO AND BOILING-POINT OF A THERMOMETER.

To find the zero point of a thermometer, any small cylindrical vessel is filled with pieces of clean ice, and the thermometer inserted so that the zero marked on the stem is just visible over the ice. Holes made in the bottom of the vessel allow the water to drain off, and the thermometer is left in until the column of mercury becomes stationary. This takes about half an hour. The position of the column is then noted and gives the error of the zero marked on the stem.

Figure 1 shews the arrangement for determining the boiling-point at a given pressure. The thermometer is placed in a vessel in which water is boiling, but so that the bulb does not touch the water. The steam circulates around it, and passes, as shewn by the arrows, through an outer jacket into the air, thus insuring that the inner temperature will be that of the steam. A small water manometer may be placed at *M* to shew any small difference in pressure between the steam inside and the air outside. When the mercurial column has become stationary, its position is noted and at the same time the barometer is read and reduced to zero. The true temperature of the steam corresponding to the reduced height is obtained from the *Tables*, and the difference between this and the observed

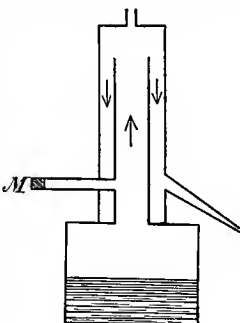


Fig. 1.

reading on the stem gives the error of the boiling-point at the pressure noted.

Thus the true temperature, corresponding to any reading of the thermometer, can easily be found.

In most thermometers sent out by reliable makers, the zero point is generally correct within a tenth of a degree, though changing very slightly from time to time; while the boiling-point may be in error as much as half a degree, and changes every time the thermometer is heated up to or above 100° C.

In connection with the reading of temperatures it may be noticed that, in a great many experiments where the thermometer is used, it is a *difference* of temperature that is required: in this case, an ordinary thermometer may be used throughout the experiment, as its errors to a large extent disappear. This is notably the case in finding specific and latent heats. And when two or more thermometers are used for different measurements in the same experiment, it is evident they should be compared with one another before and after the experiment.

II. CALIBRATION OF A TUBE.

To examine the bore of a thermometer, the apparatus shewn in the adjoining figure may be used.

A thread of mercury of any desired length is detached from

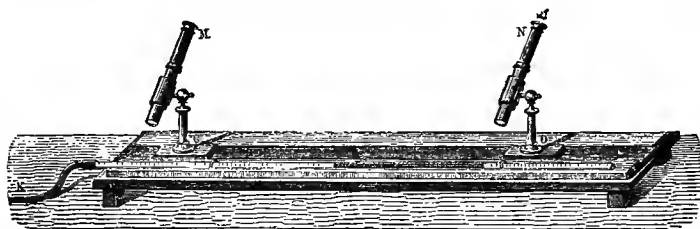


Fig. 2.

the column, and the spaces it occupies at various points in the tube noted. Usually, however, before the thermometer is made,

a number of capillary tubes are examined by means of this instrument, and only a tube of uniform bore chosen for the manufacture of the thermometer.

To examine the bore of a capillary tube, open at both ends, a small thread of mercury is introduced by attaching a piece of rubber tubing *K* to one end, and either allowing the mercury to run in, or else to be drawn up by suction.

The tube is then placed under the microscopes *M*, *N*, which are supported on two movable stands *C*, *D*, alongside of a scale graduated into millimeters; and the lengths occupied by the thread in different positions are then measured, by means of this scale, and the micrometer glasses in *M* and *N*.

A curve may be constructed, of which the ordinates represent the lengths occupied by the same thread, and the abscissæ are the distances of one end of the thread from some fixed mark on the tube, which may be taken as the origin of coördinates.

Most fine tubes will be found slightly conical in bore.

A tube of rather large bore is calibrated by simply pouring into it equal volumes of mercury determined by weighing, and making corresponding marks on the tube.

In many cases, one may wish to calibrate a large tube which has already divisions on it indicating equal distances. The readiest and most accurate way to do this is to fill the tube with mercury to a known mark; and then, allowing the mercury to flow out for a small number of divisions, weigh this quantity; again allow the mercury to flow out for a small number of divisions (not necessarily the same as before), weigh, and repeat the operation as far as is desired. Then a curve can be constructed which will enable one to read off by interpolation the volume corresponding to any division; the fixed mark being the origin of co-ordinates, the numbers of divisions the abscissæ, and the volumes the ordinates. If necessary, the fixed mark may be chosen as one end of the tube, which will then be completely filled with mercury at the first operation.

III. COEFFICIENT OF EXPANSION OF MERCURY.

The most important constant, and one upon which nearly all others are ultimately based, in experiments with heat, is the coefficient of expansion of mercury. By this is meant the fraction which represents the volumetric increase in a quantity of mercury for each increase of temperature of 1°C .

The method used is that based upon a principle suggested by *Boyle*, and devised by *Dulong and Petit*. If a bent tube, such

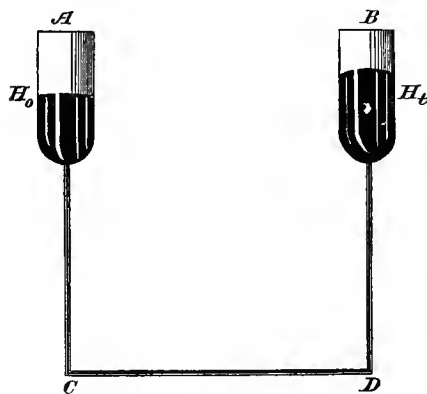


Fig. 3.

as is shewn in Fig. 3, contain in its two arms fluids of different densities, the weight of a cylindrical column of fluid in one arm standing upon any base is equal to the weight of a cylindrical column in the other arm standing on an equal base in the same horizontal plane.

By filling the tube with mercury, and surrounding *AC* and *BD* with envelopes so that *AC* may be cooled to zero and *BD* heated to the temperature of steam, we obviously can calculate the coefficient of expansion by observing the heights of H_0 and H_t from the common level below. For, conceive two small cylindrical columns standing on equal bases, whose heights from *CD* are measured. The mass of these two columns being the same, since their weights are the same, their volumes are simply to one another as their heights, since they stand upon equal bases.

Let H_0 = height of cold column.

H_t = height of hot column.

V_0 = volume of small cylindrical column in *AC*.

V_t = volume of small cylindrical column in *BD*.

t = difference in temperature.

Then

$$\frac{H_0}{V_0} = \frac{H_t}{V_t},$$

$$\therefore \frac{H_0}{H_t} = \frac{V_0}{V_0(1 + Kt)},$$

since the relation $V = V_0(1 + Kt)$ determines the coefficient of expansion K .

$$\therefore K = \frac{H_t - H_0}{H_0 t}.$$

The apparatus used commonly for the determination of this constant is made by surrounding AC and BD with two brass jackets, one of which holds ice, and the other has a tube leading into it which supplies steam from an adjoining boiler. The measurements must not be taken until the temperature of the hot column, as indicated by a very accurate thermometer, is steady. The ice must be frequently renewed, as it melts rapidly.

The greatest difficulty in the ordinary form of the apparatus is to determine accurately the mean temperature of the heated column. Measurements of heights must be made as exact as possible; a small error means a great deal when the experiment is performed with ice and steam, the difference in level being in that case not quite a centimeter. Hot air or boiling oil may be used instead of steam; but the latter is exceedingly dirty and gives off disagreeable odours: it has the advantage, however, of giving a greater range of temperature.

IV. WEIGHT THERMOMETER: CUBICAL EXPANSION OF A GLASS ENVELOPE.

The absolute expansion of mercury being assumed, the volumetric expansion of a glass envelope can easily be determined by filling it with mercury at zero, and then raising it to some known temperature, when a certain quantity of mercury es-

capacities, representing the expansion of the enclosed mercury less the expansion of the envelope.

Theory. —

Let W_0 = weight of mercury filling the envelope at zero.

W_t = weight of mercury filling the envelope at temperature t .

k = coefficient of expansion of mercury.

δ = coefficient of expansion of glass.

Then, weights being proportional to volumes, we have

$$W_0(1 + \delta t) = W_t(1 + kt).$$

For, the left-hand side of this relation represents the internal capacity of the envelope at temperature t ; and W_t represents a quantity of mercury such that, if allowed to expand from zero to t , it would just fill the envelope at that temperature.

From this δ is found, all the other quantities being known.

Experiment. — To perform the experiment, any arrangement will suffice whereby a glass vessel can be completely filled with

mercury at zero, and then heated to some known temperature; but the following apparatus will be found very convenient.

A glass envelope, with a capillary tube attached and bent twice, is placed in a receptacle as shewn in the figure below.

The end of the capillary tube dips into a small iron vessel containing mercury, and the bulb is prevented from breakage by being surrounded with a small

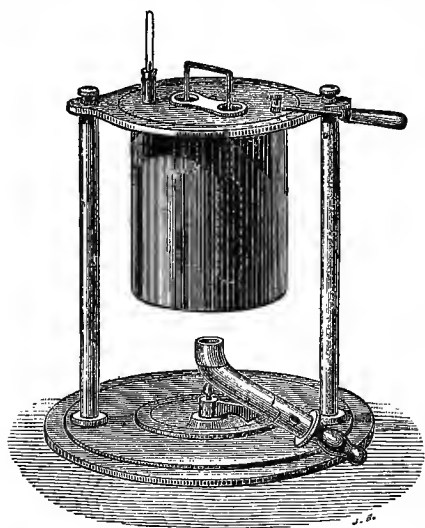


Fig. 4.

iron basket; the whole being supported in a circular brass plate with a handle attached. The outer vessel is of thick copper, and, when heated from below with a gas-burner, the air inside the bulb expands and bubbles up through the mercury in the iron cup. Then it is cooled by lifting out bodily the plate by the handle, and exposing to the cool air. This causes the air in the bulb to contract, and some mercury is then drawn in; and, by alternately heating and cooling, the bulb is finally filled with mercury. If one wishes to perform the experiment more rapidly, the outer copper vessel may be removed, and the basket holding the bulb heated directly with a spirit lamp: this, however, must be done with care, as there is danger of breakage.

Then it is surrounded with ice and cooled to zero; and, finally, water being put in the copper vessel, it is heated to the temperature of steam.

Three weighings are necessary:

1. Weight of the envelope when empty and perfectly clean and dry.
2. Weight of the envelope after heating to the temperature of steam and allowing to cool properly.
3. Weight of mercury which escapes between zero and temperature t .

These weighings give W_0 and W_t .

Precautions. — Heat slowly, and boil the mercury in the bulb to remove air and moisture. Cool slowly to avoid breakage. The experiment throughout must be performed slowly and carefully to insure success.

It is evident that the envelope, when once filled with mercury, and its elements determined, may serve as a *weight thermometer*. For, to find the temperature at any point where it may be placed, it is sufficient to fill it at zero and then bring it to the place in question, and notice how much mercury escapes. Then, as before,

$$W_0(1 + \delta t) = W_t(1 + kt),$$

and the quantities W_0 , W_t , δ , k being known, t is found.

The weight thermometer is theoretically a perfect instrument; for, by its use, we determine temperatures in terms of *masses*, which can always be measured with great accuracy; the great objection to its general use is the length and tediousness of the process.

V. COEFFICIENT OF EXPANSION OF DRY AIR AT CONSTANT VOLUME.

In this experiment dry air is taken as the type of a perfect gas, and its expansion determined from the increase of pressure necessary to keep its volume constant when its temperature is raised.

The apparatus used is that devised by Regnault, and is represented in Fig. 5.

The air to be operated upon is inclosed in a glass bulb *A*, which is placed in a boiler clamped to a stand at *K*. The water can be run off from this boiler by a small stopcock below, and replaced, when necessary, by ice, without disturbing the bulb. A special perforated jacket of tin surrounds the bulb inside to insure a constant temperature when steam is generated. A capillary tube of copper, attached to the bulb, is connected at *m*, *n* by means of a three-way stopcock, with a drying apparatus *H*, and an exhausting pump *P*, and also with a manometer of mercury, which is itself provided with a three-way tap at the bottom of the tube *BC*. These three-way taps have openings in the form of a T, and thereby enable one to make communication between any two of three sources, or to shut any two off, or all three; the directions in which the openings run are shewn on the outside by small lines. The drying tubes contain pumicestone moistened with sulphuric acid.

The connections are all made so that no leakage takes place, and the manometer tubes are filled with mercury until they both stand at the same level *a*, everything being in communication with the air. Then the three-way tap at the bottom of *C* is

closed, so that the tube *BC* does not communicate with *DE*. The process of drying the air then commences. Water is placed in the boiler and steam generated, and then exhaustion is made by *P*, through the drying tubes; finally air is allowed to enter by means of a central tap placed at the bottom of *P* which makes direct connection with the air. This operation is

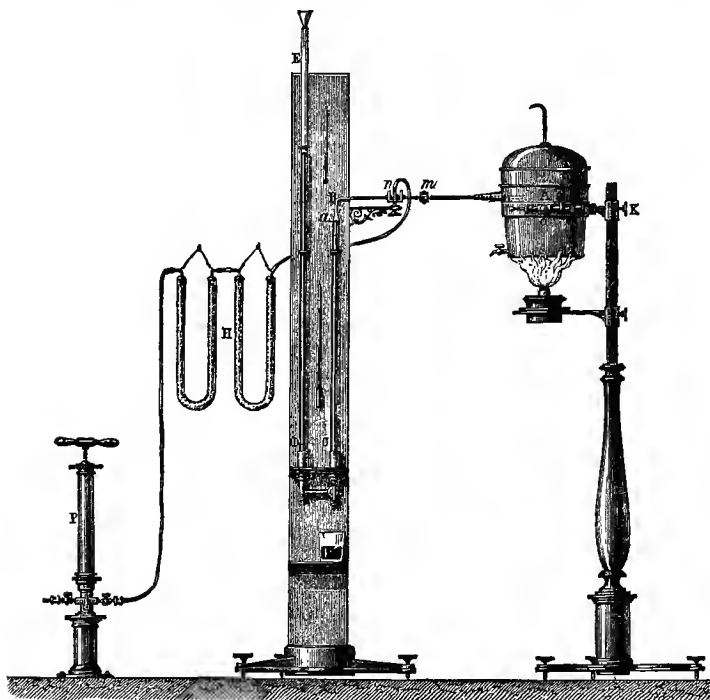


Fig. 5.

repeated a number of times, the water still boiling, and, at length, when the air is dry, the tap at *C* is opened, and the apparatus throughout brought to atmospheric pressure. Then the hot water in the boiler is run off, replaced by ice, and the air in *A* cooled to zero; and when the ice has been in about twenty minutes, the stopcock at *n* is turned so that *A* com-

municates only with the manometer. The barometer is then read, and the levels of the two columns of mercury in the manometer (which should be the same) noted.

There is now a volume of dry air inclosed at zero, and at a known pressure. The next operation is to heat this air to the temperature of steam, and as it expands, driving before it the column aC , to pour mercury in ED so that it keeps the air always at constant volume, and the mercury in BC therefore always at the mark a . This second operation will last about fifteen minutes; and when the column in DE remains stationary the difference in level in the two tubes is noted and the barometer again read.

Theory.—

Let V_0 = volume of air inclosed in A at zero.

v = volume between the marks n and a .

H_0 = barometric reading when air is at zero.

H = barometric reading when air is heated.

h = difference in level in the two tubes at the end of the experiment.

T = temperature of steam when h is read.

t = temperature of room.

δ = coefficient of expansion of the glass envelope.

α = coefficient of expansion of dry air.

Then, by Boyle's law,

$$\left\{ V_0 + \frac{v}{1 + \alpha t} \right\} H_0 = \left\{ \frac{V_0(1 + \delta T)}{1 + \alpha T} + \frac{v}{1 + \alpha t} \right\} (H + h,$$

where $V_0(1 + \delta T)$ represents the internal capacity of the glass envelope at the temperature T . The coefficient of expansion of glass may be obtained from the tables; but this is only approximate, as its value depends more on the form of the envelope than on the quality of the glass, and for accurate purposes should be

determined in each case. The capacity of the bulb at zero may be found in the usual way and marked on the outside.

In the preceding formula a first approximation to the value of α is found by neglecting v . This value is then used to reduce the expression $\frac{v}{1 + \alpha t}$ in the second approximation. The experiment may also be performed in the reverse order, by filling the bulb with hot air at atmospheric pressure and then cooling it to zero: in this case α represents a contraction, and the formula will be altered correspondingly.

Precautions. — In surrounding the bulb with ice, after it has been raised to the temperature of steam, care must be taken not to place ice on the metal socket of the glass too suddenly. It is better to run off the hot water, then cool the metal boiler with cold water, and finally the bulb. The stopcock at C must be closed when drying the air, otherwise the mercury will run over into the drying tubes and bulb. When finished, the apparatus should be opened to the air and the mercury in the tubes let down to the same level. The use of the three-way tap at C and at n must be fully understood.

VI. COEFFICIENT OF EXPANSION OF DRY AIR AT CONSTANT PRESSURE.

The mode of operation in this experiment is similar to that in the previous one, the only change being that when the dry air is heated from zero to the temperature of steam the mercury is allowed to flow out until the two columns are at the same level. The preceding formula then applies, h being zero, and v on the left-hand side becoming v' on the right-hand side.

The apparatus used is shown in Fig. 6.

The tubes BC , ED are inclosed in a water jacket to insure constancy of temperature, the bulb A is placed as before in a boiler (not shewn), and the connections at m and n lead by a three-way tap to drying tubes and an air-pump.

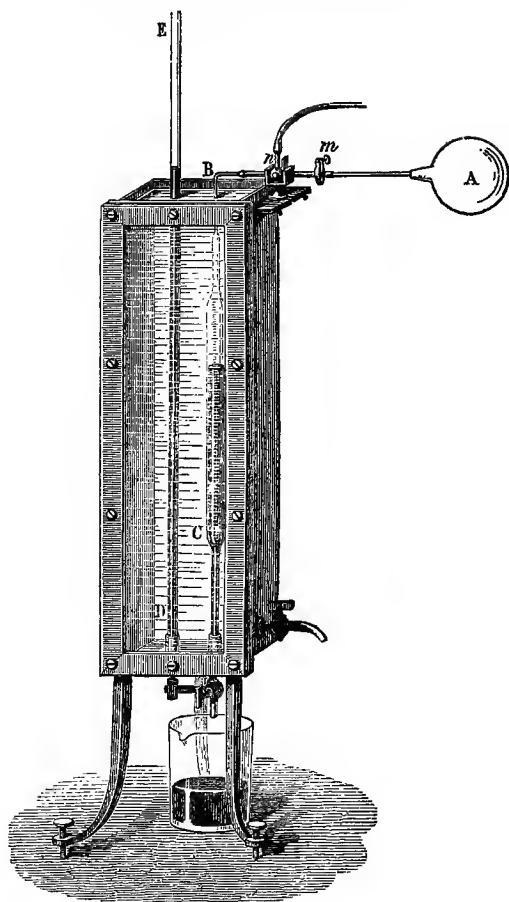


Fig. 6.

Theory. —

Let V_0 = volume of bulb at zero.

v = small volume to a chosen mark.

v' = volume when BC is lowered.

H_0 = barometric pressure at zero.

H = barometric pressure at temperature of steam.

T = temperature of steam.

t = temperature of water.

δ = coefficient of expansion of glass.

α = coefficient of expansion of air at constant pressure.

Then, by Boyle's law,

$$\left\{ V_0 + \frac{v}{1 + \alpha t} \right\} H_0 = \left\{ \frac{V_0(1 + \delta T)}{1 + \alpha T} + \frac{v'}{1 + \alpha t} \right\} H.$$

In this v' will be much greater than v ; both are found by filling the tube BC with mercury to the tap n , and then allowing it to run out to the marks which define v and v' ; the volumes are then inferred from the weights of the quantities of mercury which escape. First and second approximations are made as before, and the general precautions are the same as in the former experiment.

VII. COEFFICIENT OF EXPANSION OF DRY AIR.—THIRD METHOD.

In this experiment dry air is cooled from the temperature of steam to zero, and both volume and pressure change. A small cylindrical bulb of glass with a bent capillary stem is used, and filled with dry air by being placed in the apparatus for determining the boiling-point of a thermometer (see Fig. 1), in connection with a drying tube and an exhausting air-pump. When filled with dry air by continual exhaustion and heating, the end is sealed up, after detaching the drying-tube a few seconds to allow it to come to atmospheric pressure. It is then placed, when cool, in the apparatus below (Fig. 7).

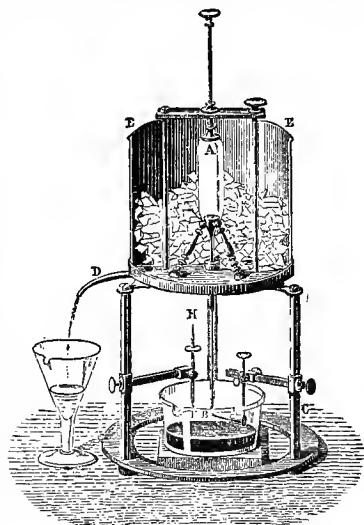


Fig. 7.

AB is the glass envelope inverted in a trough of mercury and surrounded with melting ice placed in a vessel shewn in section

by *EE*. *D* is a small outlet for the melted ice. At *C* is an arm which can be moved up and down, and to this is attached a small pipe-shaped piece of metal whose bowl is filled with wax; this can be pushed in against the end of the capillary tube, and enables one to close it under the mercury. *H* is a screw which can be adjusted to touch the surface of the mercury.

Order of the Experiment. — 1. Determine the coefficient of expansion of the glass bulb in the same manner as given in Exp. 4.

2. Place it in the apparatus shewn in Fig. 1, and connect with drying-tubes and an air-pump; and when the air in it is completely dry, allow it to come to atmospheric pressure by separating it from the drying-tubes (while the steam is still circulating around it) for a few seconds. Then seal the end with a blow-pipe, at the same time noting the barometric pressure.

3. Arrange as in Fig. 7, and break off the point underneath the mercury. This is done most readily by making a small file mark near the end before inverting it, and then with a pair of pliers breaking it off under the mercury, at the same time holding firmly the stem at *B*.

The mercury then rises into the capillary tube, and on filling *EE* with ice, it finally takes up a permanent position. Then the screw *H* is moved down until its lower point touches the surface of the mercury, and the bowl of soft wax is pressed in against the end of the tube, sealing it up, and at the same instant the barometer is read. The ice being next carefully removed, and everything allowed to come to the temperature of the room, the height from the top of *H* to the top of the column of mercury is measured with a cathetometer, and then, finally, the length of the screw *H*. The bulb is then removed, and the mercury taken from it and weighed.

Theory. —

Let V_0 = volume of mercury filling the bulb at zero.

δ = coefficient of expansion of the glass bulb.

H_0 = barometric reading when the end is sealed up under the mercury.

H = barometric reading when sealed at the temperature of steam.

T = temperature of steam.

h = height of column.

v = volume of mercury which runs into bulb when cooled.

α = coefficient of expansion of dry air.

Then, by Boyle's law,

$$(V_0 - v)(H_0 - h)(1 + \alpha T) = V_0(1 + \delta T)H,$$

from which α is determined.

Precautions. — Care must be taken in determining V_0 and δ . Some special apparatus should be used for filling the bulb with mercury, either by the application of heat or by means of a good air-pump and a three-way tap. The air must be well dried. In finding v by weighing the mercury, some difficulty will be found in getting it out of the bulb; it is well to weigh the bulb previous to the experiment, and then weigh bulb and mercury at the end of the experiment, checking, if possible, by weighing separately the mercury. A small error is introduced by the wax and the piece broken off; but these can be allowed for without any difficulty. The values of H_0 , H , h are supposed to be taken for some standard temperature, usually zero.

VIII. THE AIR THERMOMETER.

The principle of the air thermometer is that a perfect gas, such as dry air, if kept at constant volume, will have its pressure increased as its temperature increases: the constant of increase being 0.003665 for each degree centigrade. Any of the preceding instruments (Figs. 5, 6, 7) may be used as an air thermometer, assuming the value of α . The most convenient apparatus is one similar to that of Exp. 5. The air thermometer itself is a glass bulb, or (for very high temperatures) a cylindrical

or spherical reservoir of *Bayeux* porcelain, with a long capillary tube of copper, so that it may be carried, if necessary, some distance from the apparatus. This thermometer is connected to a manometer, drying apparatus, and air-pump, just as in the former experiments. Then, having obtained dry air, it is cooled to zero, and the barometer read: the two columns of the manometer being at the same level. Then the stopcock leading to the air-pump is closed, and the bulb is placed where the temperature is desired, and as the air in it expands, the volume is kept constant and the increase of pressure and barometric reading taken. Then the formula of Exp. 5 applies:

$$\left\{ V_0 + \frac{v}{1 + \alpha t} \right\} H_0 = \left\{ \frac{V_0(1 + \delta T)}{1 + \alpha T} + \frac{v}{1 + \alpha t} \right\} (H + h),$$

where α is now assumed, and T , the unknown temperature, is to be found. This process can be used to graduate a mercurial thermometer: the reservoir of air is raised from zero to temperatures gradually increasing, and the temperatures calculated from the formula are compared with the indications of the mercurial thermometer.

For rough calculations v may be neglected, when it is very small, and also δ ; then it is not necessary to know V_0 , and we get the temperature from the simple relation

$$V_0 H_0 (1 + \alpha T) = V_0 (H + h),$$

and

$$T = \frac{(H - H_0) + h}{0.003665 H_0}.$$

The essential precaution in the use of the air thermometer is to have the air perfectly dry: this is insured by the use of a good Sprengel or Geissler pump in connection with a reservoir of sulphuric acid.

The porcelain reservoir is used for the determination of temperatures beyond that at which glass begins to melt.

IX. FAVRE AND SILBERMANN'S CALORIMETER.

This is in reality a large thermometer, consisting of a reservoir *A* (Fig. 8), which contains about fifty pounds of mercury and which has a graduated stem *CD* with a fine bore into which the mercury of the reservoir when heated can expand.

A microscope *L* provided with a micrometer glass enables one to estimate the divisions on *CD* to twentieths of a millimeter. A special contrivance at *B*, with a handle, is used to put pressure

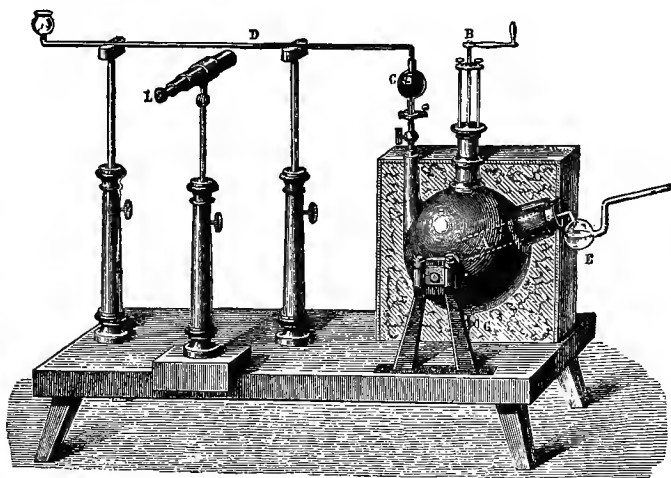


Fig. 8.

on the interior of the reservoir and move the thread of mercury to any desired point; and there are two or three openings in *A* in which heated bodies may be placed and thereby give out their heat to the mercury in the reservoir.

The instrument is first graduated by taking a known weight of water at a known temperature, and inserting it, as shewn at *E*, and noticing the number of divisions over which the thread *D* moves, as the heat from this water is imparted to *A*.

The theory of the instrument is simple.

Let M = mass of water introduced.

T = temperature when introduced.

θ = its temperature when the thread is stationary.

n = number of divisions over which the thread has moved.

Then $\frac{M(T-\theta)}{n}$ = value of one division in units of heat
 $= c.$

To measure, then, a specific heat of a liquid, for example,

Let P = mass of liquid introduced.

T = its initial temperature.

θ' = its final temperature.

x = its specific heat.

m = number of divisions over which the thread moves.

Then $Px(T-\theta') = mc,$

from which x is found.

The instrument can be used for measuring specific heats of liquids, heats of combustion, and even specific heats of solids. Care only must be taken in determining the constant c , which should be found from the mean of a number of observations.

X. LATENT HEAT OF STEAM.—REGNAULT'S METHOD.

This is essentially a method of mixture, the apparatus for performing the experiment being shewn in Fig. 9.

B is a boiler in which steam is generated either at atmospheric or other pressures. The steam is led from this boiler into a brass globe G placed in a calorimeter C containing a known weight of water. The handle H when turned in a certain direction allows the steam to pass, by a peculiar arrangement of two hollow tubes working within one another, into G ; when turned in all other directions the steam escapes through

an exhaust pipe *E* and condenses in a vessel below. The pipes *P, P* are used when pressures greater than atmospheric are needed; they lead to a brass box, which can be connected with a mercurial manometer, or gauge of any kind to indicate the pressure at which the steam is generated. The steam passes then into *G*, condenses, and gives out heat to the surrounding

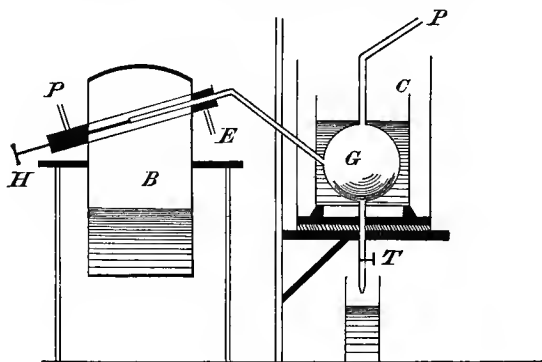


Fig. 9.

water in the calorimeter, which rises correspondingly in temperature. After a certain time has elapsed, the steam is shut off by turning *H*, and then, waiting a few minutes to allow the condensed steam in *G* to come to its lowest temperature, the tap *T* is opened and the water of condensation collected and weighed or estimated by volume. The temperature of the water in the calorimeter, at the beginning and end of the experiment, is taken with a thermometer reading directly to fifths or tenths.

Theory. —

Let W = weight of water, together with the water equivalent of the calorimeter.

t = its temperature.

w = weight of condensed steam.

T = temperature of the steam.

T' =temperature of condensed steam when run out from the calorimeter.

θ =final temperature of the water in the calorimeter.

x =latent heat of the steam.

Then, by the equivalence of heat lost and gained,

$$W(\theta - t) = wx + w(T - T').$$

For the left-hand side of this relation represents the heat absorbed by the water and calorimeter; and the right-hand side is the heat given out by the steam in condensing, together with the heat given out by the condensed steam in falling from the boiling-point to its final temperature when run off.

Precautions. — 1. The greatest difficulty, in such an experiment as this, is to determine accurately the water equivalent of the calorimeter, which may either be found by an independent experiment, using the method of mixture, or determined in conjunction with the latent heat of steam by the method of successive approximation.

2. The water in the calorimeter should be in the first place as cold as possible.

3. The steam, in passing into the calorimeter, is made to heat the water as much above the temperature of the room as it previously was below. This is to equalize radiation and absorption. If the temperature of the room be above 18°C. , then the initial temperature of the water should not be more than 8°C. ; otherwise the final temperature of the water will be so high that rapid evaporation takes place from its surface.

4. A series of temperatures should be taken as the condensed steam is running out, and the mean in some way accurately found.

5. The water should be well stirred just before the steam is let into the calorimeter, and its temperature carefully read. Stirring should be kept up throughout the experiment, especially if the surface of the water is near the pipe which enters

into G , for in that case the surface water becomes very hot and gives off steam.

6. Care should be taken not to let the boiler run dry, and when H is closed, to open the exhaust pipe E , or the outlets P , P .

XI. LATENT HEAT OF STEAM. — BERTHELOT'S METHOD.

Although the preceding method can be applied to calculate the latent heat of steam and other vapours at different pressures, yet it necessitates large quantities of liquid, and the errors of experiment are not easily corrected. Berthelot's arrangement is more simple, avoiding thereby several sources of error; it can be used for any volatile substance, requires but a small quantity of liquid with which to operate, and gives accurate and rapid results.

It consists of a calorimeter C (Fig. 10) of special construction, filled with water or other non-conducting material, and covered with felt, and an inner vessel c in which a known quantity of water is placed.

The boiler of the preceding experiment is replaced by a bottle-shaped vessel G , in which the liquid to be examined is placed; and this, being heated from below, sends vapour, as shewn by the arrows, through a tube AD into a receptacle B , where it condenses and gives out its heat;

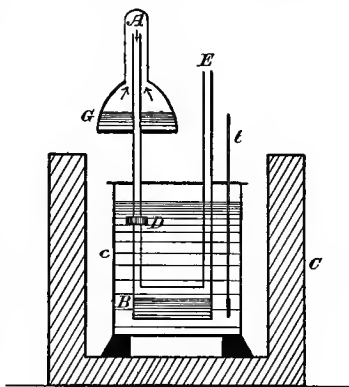


Fig. 10.

owing to the compact form of the apparatus, no heat is lost in the passage from G to B . At D is a detachable clamp by means of which one can separate the boiler from B ; and a spiral tube generally runs from D to B , so that condensation is more easily effected. The exit at E insures that vaporization

takes place at atmospheric pressure. The boiler and receptacle, with attached tubes, may be made of glass or metal; if of glass, then the instrument can be used for almost any volatile liquid, and has the advantage of being transparent.

The water equivalent of the calorimeter in this case is easily found by detaching *B* and weighing it; and the heat from the burner which is placed under *G* can be prevented from heating the water in *c* by placing over the lid of the apparatus a layer of asbestos.

The thermometer *t* should be graduated in tenths of degrees.

To perform the experiment, unfasten the clamp *D*, weigh *B* and connecting tubes, and determine its water equivalent; then invert *G*, pour in the liquid to be heated, and arrange as in the figure. The method being one of mixture, the calculation is the same as in the preceding experiment.

XII. LATENT HEAT OF VAPORIZATION OF VOLATILE LIQUIDS.

For liquids which evaporate rapidly at ordinary temperatures a reverse process devised by Regnault may be used.

It consists in evaporating a quantity of the liquid at a low pressure in a vessel surrounded with water, and from the diminution in temperature of this water calculating the heat necessary for vaporization. The apparatus used is similar to that shewn in Fig. 11.

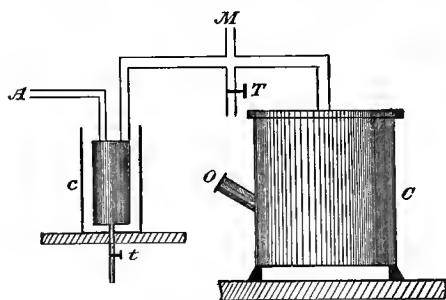


Fig. 11.

The liquid to be evaporated is poured into a calorimeter *C* through a tube at *O*, which is then closed with a tight-fitting screw cap. The calorimeter portion consists of three vessels: an inner

closed one in which the liquid is placed, a second to hold the water to be cooled, and an outer one to prevent direct radiation or absorption of heat. A tube leads from the innermost closed vessel to another small calorimeter c , and then to an air pump connected at A . The tube M is connected with a pressure gauge or manometer.

A freezing mixture is generally placed in c , so that most of the vapour produced by evaporation in C condenses, and can be collected again by opening the tap t . A stirrer and thermometer are also necessary.

Theory. —

Let W = weight of water in the calorimeter.

w = water equivalent of the calorimeter.

T = initial temperature of the water.

t = final temperature of the water.

P = weight of liquid.

c = its specific heat.

x = latent heat of vaporization.

θ = temperature of vaporization.

Then

$$(W + w)(T - t) + Pc(T - \theta) = Px,$$

from which x is found.

In addition to the term Px there will be another term (due to the vapour leaving the calorimeter becoming heated), which would involve the specific heat of the vapour; but it will generally be very small. There is also a slight uncertainty in the determination of the temperature of ebullition (θ).

XIII. WEIGHT OF A LITER OF DRY AIR AT NORMAL PRESSURE AND TEMPERATURE.

The method employed is that of Regnault, in which two hollow glass globes are blown as nearly as possible identical in every respect, the capacity of each being about seven liters. One

of these is sealed up permanently, and the other is provided with a stopcock. They are suspended by hooks from the scale pans of a large balance and are brought into a state of equilibrium by the addition, if necessary, of small weights; if a very accurate experiment is required, they are next suspended in water, and due allowance made for any difference in their external volume shewn by the unequal upward pressures.

In this way one obtains two bodies which displace equal amounts of air, whatever be its pressure, temperature, and hygrometric state.

The globe provided with a stopcock is then removed and placed in connection with an air pump and mercurial gauge, and drying apparatus, as shewn in the adjoining figure.

G is the globe, which may be surrounded either with water of a known temperature or, better, with ice. T' is the stop-

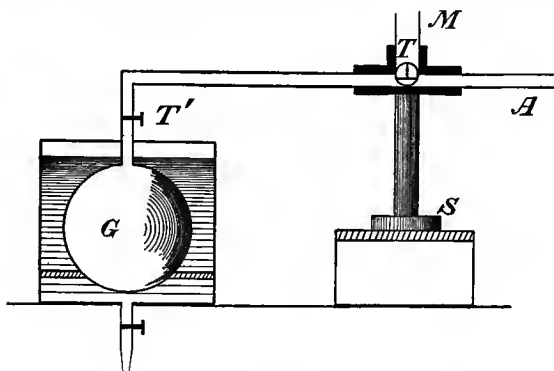


Fig. 12.

cock, which when open makes communication with a three-way tap T attached to a stand S . At A is an outlet leading to a drying apparatus, gauge, and air pump.

When T' and T are so adjusted that the globe G communicates with A , it may be filled with perfectly dry air at a known temperature, usually zero: it is then left in communication with the air for a few seconds and the barometric reading taken.

After assuming the temperature of the room, it is again put

in the scale pan, and equilibrium established with the compensating globe and small weights. Once more it is removed, placed in connection with the air pump and gauge, and exhausted as completely as possible; and the indication of the gauge along with the barometric reading taken. Finally, after closing T' , it is placed on the scale pan, and equilibrium restored by the addition of certain new weights, which must represent the weight of the dry air removed from the globe.

Then it only remains to determine the internal capacity of G at zero, which is done by taking the exhausted globe and placing it, by means of T , in connection with the outlet M , without opening T' . A tube being now led from M into a vessel of water, T' is opened, and the water rushes from M into G and almost completely fills it. The opening in the top of the globe is usually so small that it would take some time to fill it in any other way.

Theory. --

Let V_0 = internal capacity of the globe at zero.

H = mean of the two barometric readings taken.

h = reading of the gauge when the globe is exhausted.

P = weight necessary in the final weighing to produce equilibrium.

Then, by Boyle's law, the volume of air removed is

$$V_0 \frac{H-h}{H},$$

and therefore the weight of a liter of air at 0° C. and 760 mm. is

$$\frac{P}{V_0} \cdot \frac{760}{H-h}.$$

Precaution. — Care must be taken, when determining the capacity of the glass globe, not to allow the water, by any mistake in turning T , to get into the air pump through A .

It is evident that the above method can be readily applied to the determination of the densities of gases.

XIV. DETERMINATION OF THE HYGROMETRIC STATE OF THE AIR.

The hygrometric state of the air will be known when we know the quantity of water vapour in a given volume of air, and also the pressure which this vapour exerts; but the determination of either of these, although simple in theory, is attended with great practical difficulties.

The hygrometric coefficients may be found by

(a) *The chemical hygrometer.*

This consists simply in passing a known volume of air through drying tubes which are weighed before and after the experiment.

The weight of vapour in a given quantity of air is thus found, and if we wish the pressure which this vapour exerts, we may find it in the following way.

Let w = weight of water in grams.

v = its volume in liters.

t = its temperature.

x = its pressure.

Then, since 1 liter of air at 0° C. and 760 mm. pressure weighs 1.293 grams, and since water vapour is .622 times denser than air, we have

$$w = \frac{vx \times 1.293 \times .622}{(1 + \alpha t) 760},$$

where

$$\alpha = .003665.$$

And

$$\therefore x = 945 \frac{w(1 + \alpha t)}{v}.$$

Conversely, if we know x , we can find w .

If we require to find the weight of a given volume of moist air,

Let V = volume in cc.

H = barometric pressure.

f = pressure of vapour of water in it, found from the dew-point and the tables.

t = temperature of air.

$\alpha = .003665$.

Then the weight of dry air in V

$$= V \times .001293 \times \frac{1}{1 + \alpha t} \times \frac{H - f}{760}.$$

And the weight of vapour

$$= .622 V \times .001293 \times \frac{1}{1 + \alpha t} \times \frac{f}{760}.$$

Therefore the weight required

$$= \frac{V \times .001293}{(1 + \alpha t) 760} \left\{ H - f + \frac{5}{8} f \right\} = \frac{V \times .001293}{760(1 + \alpha t)} \left(H - \frac{3}{8} f \right).$$

To perform the experiment (a), a set of drying tubes is attached to an aspirator (shewn in section in Fig. 13), consisting of two reservoirs R, R , capable of holding water, and at the same time of rotating about a fixed horizontal axis AB . By means of the tubes C, D , the water falling from the upper reservoir into the lower drags air through the opening at A and forces it through the exit E , as in the figure. It will also be seen that when the lower reservoir is full, the instrument can be inverted, and the same process repeated without disturbing any connections made at A or E . Any required volume of air

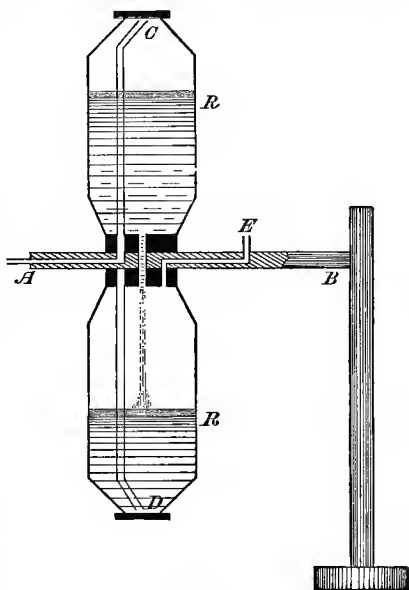


Fig. 13.

can then be operated upon in a short time. The drying tubes are, of course, connected at *A*.

The volume of air dragged through the aspirator at one operation may be taken approximately to be the capacity of the reservoir, which is supposed to be initially full of water. If a very accurate experiment is required, account will have to be taken of the change in pressure owing to the final air in the aspirator being saturated with water vapour and at a different temperature from that which came to the drying tubes.

It is preferable to have three drying tubes, two for the absorption of the moisture from the air upon which one operates; and the third, placed next the aspirator, to absorb any moisture which may chance to get back from the aspirator, which is filled with water: the two outside ones are then weighed before and after the experiment.

Tubes filled with calcium chloride or pumice stone and sulphuric acid may be used, and instead of making them air-tight with paraffin or wax, it is better that they should be provided with perforated glass stoppers: they can then be opened or closed at will.

(b) *Condensation hygrometers.*

The determination of the hygrometric state of the air by chemical means is slow, and does not give accurate information of the variations in humidity during short intervals of time: it is really a mean of the states which the air had during the time of the experiment. More minute and rapid information is obtained by the hygrometers of condensation, which are arrangements whereby the temperature inside a vessel is so lowered that dew appears on the outside owing to condensation of moisture from air in its vicinity. A glass of ice water on a hot day is the simplest form of hygrometer; and the formation of dew on the earth when it loses its heat by radiation into space shews how the earth itself, under favourable circumstances, becomes a huge hygrometer.

1. *Regnault's hygrometer.*

In this instrument, which is shewn in section in Fig. 13 (a), there are two small tubes *A* and *B*, the upper portions being of glass, the lower, caps of silver or silvered copper: these are attached to a stand so as to be near one another. In *A* is placed a cork with a thermometer *T'*; in *B* a cork with a thermometer *T* and two glass tubes *C*, *D*. When in use *B* is partially filled with alcohol or ether, and the tube *C* connected with an aspirator; or else air is forced through the tube *D*. The evaporation of the alcohol or of the ether thus produced lowers the temperature of the metal cap, and dew is seen to form on the outside. The temperature then shewn by *T* is called the *Dew-point*. At the same instant *T'* is read, and the two temperatures *T*, *T'* determine the humidity.

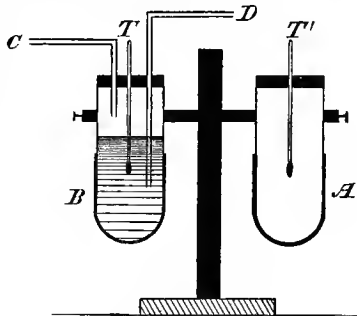


Fig. 13 (a).

Let p = weight of moisture in a given volume of the surrounding air.

P = weight of moisture in the same volume of the surrounding air, supposed to be saturated.

f = pressure of moisture in p .

F = pressure of moisture in P .

Then by what has already been shewn,

$$\frac{p}{P} = \frac{f}{F}$$

Now from Regnault's tables of vapour tension, f and F are known when we know T and T' . Hence the relative humidity is at once given by these two temperatures. Usually, for practical

purposes, this ratio $\frac{f}{F}$ is expressed in terms of a unit of saturation taken to be 1 or 100. Thus, if $T=6^{\circ}\text{C.}$ and $T'=15^{\circ}\text{C.}$, the humidity would be .55, or 55.

Precautions.—In finding the dew-point, the aspirator is adjusted so that the dew appears and disappears within small limits of temperature, and a mean taken. The two caps should present the same appearance and be placed in a suitable light. The thermometers may be read by a telescope placed at a distance. In warm weather alcohol may be used in B ; but in winter, when the dew-point sometimes falls below freezing point, ether must be used.

2. Alluard's hygrometer.

This is an improved form of Regnault's, in which the two metal caps are replaced by a brass box A plated with gold, and a piece of the same material B (Fig. 14) placed near it, but not in contact, so that a comparison can be readily made between the two surfaces, and the presence of dew on A at once detected. Inserted in the box A is a thermometer t , which can be read if necessary through the glass window in A . Another thermometer t' gives the temperature of the surrounding air. The liquid to be evaporated is poured in at E , which is provided with a cork to prevent evaporation, and two metal

tubes D , C lead into the box, the former just entering it, and the latter running to the bottom. Two stopcocks are provided at F , G .

To use the instrument, ether is poured in at E and the cork replaced; the stopcocks F , G are opened, and a tube leading from an aspirator attached to I . The dew is made to appear

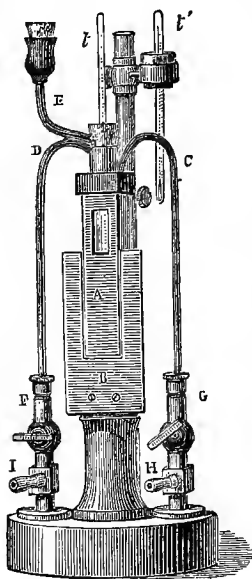


Fig. 14.

and disappear a number of times, and a mean temperature taken, t' being read at the same time. When finished, one may remove the ether by blowing into I , and having another tube leading from H into a small bottle.

The instrument is compact and can easily be carried about in a box; and instead of using an aspirator one may blow through the tube attached to H in order to induce evaporation of the ether.

(c) *Regnault's experiment.*

This is to shew that in a given space occupied by a gas a liquid will produce a quantity of vapour whose weight and ten-

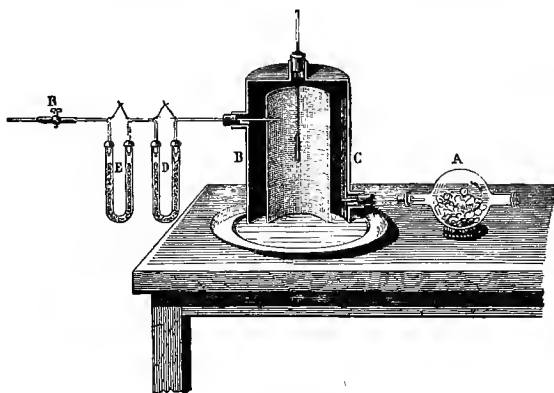


Fig. 15.

sion are the same as if the space were a vacuum; or, in other words, vapour from any liquid forms in presence of a gas just as in a vacuum, and its pressure is added to that of the gas. That the latter is true may be readily shewn by the apparatus used for proving Boyle's law (Exp. 12, Elementary Course); and the following experiment enables us to shew that the *weight* of vapour formed in presence of a gas is equal to that produced *in vacuo*.

Fig. 15 shews the apparatus. A glass bulb A , open at one end, is filled with pieces of sponge saturated with water; the

other end fits into a closed brass vessel BC , with a sieve inside covered with wet pieces of cotton waste; another tube leads from the inside of the sieve, through two drying tubes D , E , to an aspirator, such as is used in the previous experiments. When the aspirator is started, air is drawn through A , filters through BC , and thereby becomes thoroughly saturated with water vapour; it then deposits its moisture in the drying tubes D , E , and enters the aspirator, where it is again saturated. A thermometer gives the temperature of the saturated air.

If now V = volume of aspirator in cubic centimeters,

F = tension of vapour, found from t and the tables,

t = temperature of vapour,

then, if the space V were a vacuum, the weight of vapour produced in it in presence of water would be

$$V \times \frac{.001293 \times .622}{760(1 + \alpha t)} \times F,$$

since a liter of dry air at 0° C. and 760 weighs 1.293 grams, and the density of water vapour is .622.

This expression should give the same result as would be obtained by getting the difference in weight of the drying tubes before and after the experiment.

The weight of vapour is then calculated from the foregoing expression, and also observed directly by weighing the drying tubes: the two results should be the same.

XV. DENSITY OF A VAPOUR (DUMAS' METHOD).

This method is one of direct weighing, a glass envelope being first weighed when empty, and then when filled with the vapour whose density is required; and finally its internal capacity found by filling it with water and again weighing.

The experiment, though theoretically simple, is attended with

many practical difficulties, and if performed with due care is a most instructive exercise for the student.

The apparatus is simple: a glass bulb with a bent stem, after being carefully dried and weighed, has a small quantity of the liquid whose vapour density is required introduced into it, and it is then placed in a vessel of water, as shewn in Fig. 16.

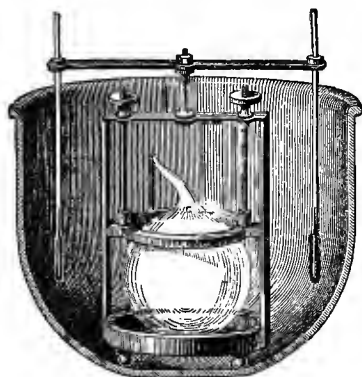


Fig. 16.

On heating this to the temperature at which the liquid boils, the air in the bulb is driven out gradually along with the vapour; and finally, when all the liquid has disappeared, the end of the stem is sealed up, and the temperature of the bath and barometric reading noted.

It is then removed, dried, and again weighed, and then, the point being broken off under water, it is filled by water rushing into the vacuum. It is again weighed in order to determine its volume.

For a very exact determination, it will be necessary to read the barometer and hygrometer when weighing, and also to take the temperature in the balance case.

Theory. —

Let p = weight of bulb (open) in grams.

p' = weight of bulb filled with vapour and sealed.

H = barometric reading when sealed.

H' = barometric reading when weighing bulb filled with vapour.

t, f = corresponding temperature of room, and pressure of vapour.

V_0 =capacity of bulb at 0° C. in cubic centimeters.

T =temperature of bath at which bulb is sealed.

k =coefficient of expansion of glass.

α =coefficient of expansion of air.

P =weight of bulb when filled with water.

H'', t', f' =corresponding barometric reading, temperature of room, and pressure of vapour.

x =density of vapour (compared with dry air at 0° C. and 760 m.m.

d =weight in grams of 1 c.c. of water at temperature t' .

In the above, usually $H=H'=H''$, and $t=t'$, and $f=f'$.

Then $p'-p$ =weight of vapour less weight of displaced air of room,

$$= V_0(1 + kT)(.001293 x) \frac{H}{760(1 + \alpha T)} \\ - V_0(1 + kt)(.001293) \frac{H' - \frac{3}{8}f}{760(1 + \alpha t)},$$

and $P-p$ =weight of water less weight of displaced air of room,

$$= V_0(1 + kt')d - V_0(1 + kt')(.001293) \frac{H'' - \frac{3}{8}f'}{760(1 + \alpha t')},$$

and on dividing one relation by another, we get x by eliminating V_0 .

These are rigorous relations from which x may be found. For ordinary purposes the following approximate formula may be used.

Let p =weight open.

p' =weight filled with vapour and sealed.

P =weight filled with water.

t, H =temperature of bath, and barometric reading when sealed.

t', H' = temperature in balance case and barometric reading when weighing.

d = density of air referred to water for t', H' .

x = density of vapour referred to air at 0° and 760 m.m.

$$\text{Then } x = \left\{ \frac{p' - p}{P - p} \cdot \frac{1}{d} + 1 \right\} \frac{H'(1 + \alpha t')}{H(1 + \alpha t')},$$

where $\alpha = .003665$.

Precautions. — 1. The bulb must be dry, clean, and carefully weighed to the tenth of a milligram.

2. Sufficient of the liquid whose vapour density is required must be put in the bulb, so that the air may be all driven out.

3. When the liquid has all disappeared, the end must be sealed and the temperature of the bath and barometric reading noted. Some difficulty will be found in doing this; with vapours such as chloroform or ether the escaping vapour may be burned, and in that way watched. On no account must the bulb be raised out of the water when nearing the end of the experiment, as the cooling causes air to be drawn in, and this is rarely all driven out again.

4. Distilled water must be used for finding the volume. If the bulb has been properly sealed, then, when the point is broken off under water, it should be almost completely filled, leaving only a small air bubble about a centimeter in diameter. For ordinary volatile vapours a water bath is used; for iodine vapour, sulphuric acid or olive oil; and if a higher temperature is required, some kind of metallic alloy may be used.

XVI. COEFFICIENT OF EXPANSION OF METALS.

The most accurate method for determining a coefficient of expansion of a metal is that devised by Ramsden, in which two microscopes, fixed in position, are provided with micrometer eyepieces and movable cross-wires, the micrometer head being graduated so that each division corresponds to a very small

fraction of an inch. The metal to be examined, made into a suitable bar, is placed under the microscopes, which are so adjusted laterally that the cross-wires coincide with certain fixed marks on the bar at a certain temperature.

As this temperature is increased or diminished, the fixed marks are seen to change position when viewed through the microscopes; and the corresponding increase or diminution in length can easily be calculated from the number of turns of the micrometers necessary to bring the cross-wires again in coincidence with the fixed marks. The method is simple, satisfactory, and, with due precaution, very exact.

Usually, for the purposes of comparing standards of length and finding their coefficients, a machine known as a *comparator* is used, which consists essentially of two microscopes as described above, and special arrangements with rollers to level the bars or to raise or lower them: some way must also be found for maintaining them at constant temperatures.

The bars may be made of any form, and fine lines ruled upon them with a dividing engine; but for standards of length they are made of square cross-section, and near each end are drilled two circular openings to the mid-depth of the bar. On the bottoms of these surfaces lines are engraved, which thus lie in the neutral axis of the bar, and are not so liable to change their position owing to flexure or strain.

XVII. SPECIFIC HEAT OF DRY AIR AT CONSTANT PRESSURE.

Regnault's experimental determination of this is the most reliable. His method, which is one of mixture, consists in heating a given mass of air to a high temperature, and then allowing it to pass at a constant pressure (nearly atmospheric) into a calorimeter containing a known quantity of water. The equivalence of heat lost and gained determines the specific heat of the air.

An apparatus similar to that devised by Regnault is shewn in Fig. 17.

A large vessel R filled with water contains an inner reservoir of about 35 liters' capacity, in which dry air is compressed by means of an air pump attached at A . A manometer at M , which may be of any kind (although for exact results a mercurial one is preferable), gives the pressure. Leading from this reservoir is a tube which passes to a heating apparatus C , runs through it in a spiral form, and emerges at the bottom, and finally leads into a small calorimeter c . A conical stopcock V regulates the flow of air into C , and by watching a small water

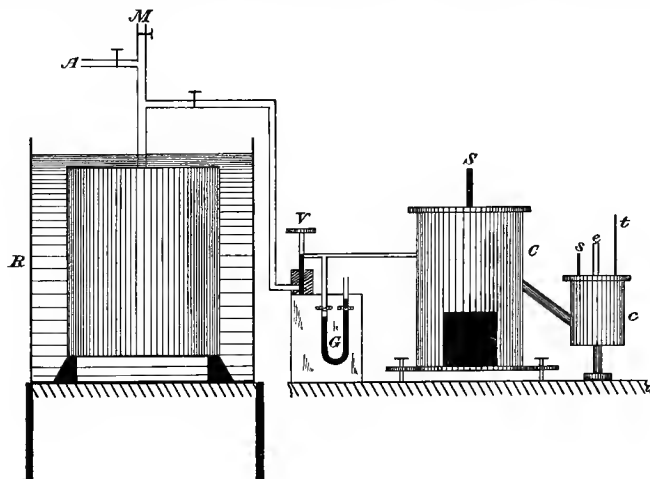


Fig. 17.

gauge G the operator can maintain a pressure which is never very much greater than atmospheric, and is perfectly steady. A stirrer S and thermometer are placed in C , and a stirrer s and thermometer t in c . The air emerges eventually at e .

Theory. —

Let H = total pressure of air in reservoir.

V = capacity of reservoir in liters.

h = pressure of barometer.

T = temperature in C .

t = initial temperature in c .

t' = final temperature in c .

W = weight of water in c .

w = water equivalent of the calorimeter.

x = specific heat of air at the mean pressure observed.

Then, after the experiment is finished, there will be a quantity of air left in the reservoir equal to V , the pressure of which is h ; and therefore by Boyle's law the volume which has escaped is

$$\frac{H-h}{h} V.$$

And, since one liter weighs 1.293 grams, weight of air

$$= \frac{H-h}{h} V \times 1.293 \text{ grams,}$$

and, by the equivalence of heat lost and gained,

$$\frac{H-h}{h} \cdot V \times 1.293 \left(T - \frac{t+t'}{2} \right) x = (W + w)(t' - t).$$

Precautions. — There are many corrections to be made in this formula if an accurate result is required. The air in the reservoir should be dry; if not, then a complicated correction is necessary, both for temperature and pressure of vapour contained in it. Evidently h will be corrected by the mean reading of G ; and the usual precautions with regard to maintaining a constant temperature in C , c , by use of the stirrers, and also to insure equal radiation and absorption in c , must be taken.

XVIII. PRESSURE OF VAPOURS FOR LOW TEMPERATURES.

(1) *Between zero and 100° C.*

The apparatus for this is that devised by Dalton and modified by Regnault: it is shewn in the following figures.

A barometer A is placed alongside of another tube B which

has an attachment at its upper end shewn in Fig. 19: this is simply a small glass globe which is detachable at *D* and provided with a stopcock *T*. A vessel *C* containing water, a stirrer, and a thermometer, surrounds the upper part of *A* and *B*.

When in use, the liquid whose vapour tension is to be examined is placed in *G*, and an air pump being attached, exhaustion is carried on until only vapour is left, and *T* is then closed. The differences in the readings of *A* and *B* at different

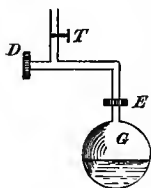


Fig. 19.

temperatures give the required pressures.

(2) *For the temperatures below zero.*

For temperatures below the freezing point of water the apparatus is slightly altered as in Fig. 20.

A and *B* are two barometer tubes: at the upper end of *B* is a glass envelope *G* in which is the liquid, so that the space above the mercury in *B* is filled with vapour. Surrounding *G* is a freezing mixture (liquid) contained in a vessel *C*. The tension of the vapour can thus be found for temperatures below freezing point, and, since the two mercurial portions are exactly in the same conditions, the tension is given directly by the difference in level between *A* and *B*.

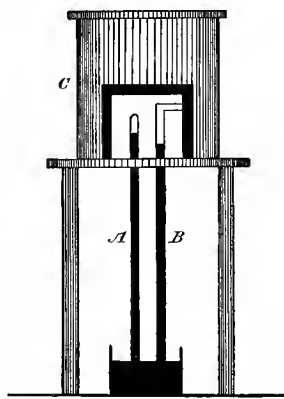


Fig. 18.

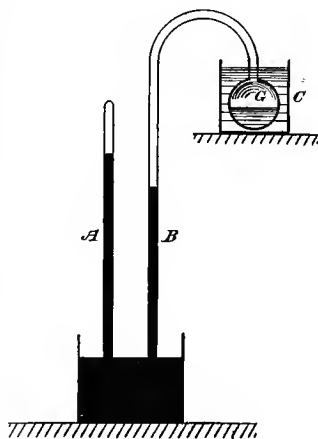


Fig. 20.

The method is exact, if one takes care to insure the absence of air in the tube B , which might be done more readily by using an arrangement provided with a stopcock, as in Fig 19.

XIX. REGNAULT'S APPARATUS FOR THE DETERMINATION OF THE PRESSURE OF VAPOURS AT HIGH TEMPERATURES.

This is represented in Fig. 21. A closed boiler B contains water (or other liquid), the temperature of which is given by the thermometer t . A large reservoir R is connected to the boiler, and a compression pump at A , with a manometer at M , completes the arrangement. A water jacket CD surrounds the

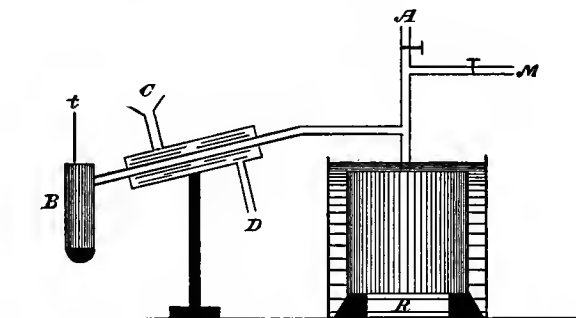


Fig. 21.

tube leading from B to R , so that the steam condenses and runs back into the boiler.

The mode of operation is to compress the air in R , determine the pressure by means of the manometer and barometer, and finally take the temperature indicated by t . Tables are then constructed, which may be graphically arranged to exhibit the relation between temperature and pressure.

ELECTRICITY AND MAGNETISM.



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ELECTRICITY AND MAGNETISM.

I. MAGNETIC LINES OF FORCE.

When a magnetized steel bar is plunged into a vessel containing iron filings, it is found on withdrawing it that these arrange themselves in tufts or bunches, not along the whole length of the bar, but round its ends, leaving the central portion quite free.

From this it would appear that the centers of attraction, or, as they are commonly called, the magnetic masses, are situated in the regions near the ends of a magnet, and it is customary to call the points in these regions to which the attractions are the greatest the *poles* of the magnet, and the straight line joining them its *axis*.

If a bar magnet be suspended so that it can move freely in a horizontal plane, it will take up a position in space pointing nearly north and south, and if it is displaced from this position, it will come back to it and will always present the same end towards the north. For this reason the end of a magnet which points to the north is called the *north* or *north-seeking* pole, and the opposite end the *south* or *south-seeking* pole. The terms *positive* and *negative* are also frequently used to designate the same poles.

Further, if the north pole of a second magnet is presented to each of the poles of the suspended one, the north pole of the latter will be repelled and its south attracted. Hence, the law of magnetic action: *Two magnetic poles of the same kind repel, and two poles of the opposite kind attract each other.*

If a bar magnet be placed on a piece of cork or wood, and the whole be then floated on the surface of water or mercury, it will be found that under the action of the earth the magnet is subjected to a pure rotation, and not to any motion of translation. As the action of the earth may be taken to be constant in magnitude and direction for points near the floating magnet, and since all the forces acting on the latter are thus reduced to a couple, we conclude that positive and negative magnetic masses are present in equal quantities in every magnet.

Again, if we attempt to separate the two poles of a magnet by breaking it into two pieces, we find that instead of accomplishing this, we only produce two magnets in place of one, and no matter into how many pieces a magnet is divided, each

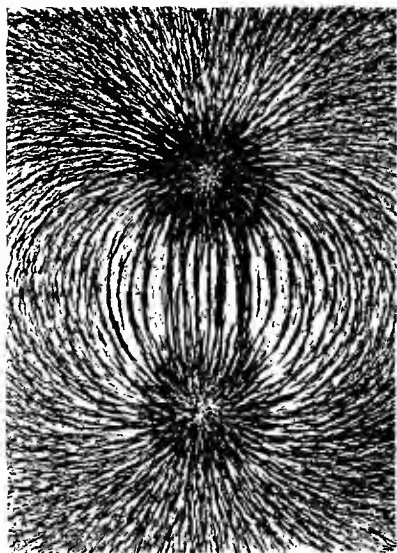


Fig. 1.

of the fragments will possess polarity just as the magnet did as a whole. We therefore conclude that it is impossible to obtain a quantity of positive magnetism which is not associated with an equal quantity of the opposite kind. Although we cannot obtain a magnet with only one pole, it is often useful in facilitating calculation to consider a single pole as existing by itself, always remembering, of course, that it is a purely mental conception.

The magnetic mass of a unit pole is defined to be such *that if concentrated in a point and placed at a unit distance from that of a similar pole, it will repel the latter with unit force.*

The *field* of a magnet is the region in which it exerts any force, and the *intensity* of a magnetic field at any point is the force exerted by the magnet on a unit pole, supposed placed at this point. If we were to imagine a free unit pole placed in a magnetic field, and moving under the forces exerted by the magnet, the path which it would describe is called a *line of force*, and it is such that the tangent to it at each point is the direction of the resultant force at that point. The lines of force for a given magnet may be plotted by placing a very short compass needle at various points in its field, and noting in what direction the needle points while in each position.

The actions of the magnet on the positive and negative poles, respectively, of such a needle will be equal in magnitude and opposite in direction, and therefore the direction which the needle takes up is that of the resultant force at the point where it is placed.

Probably one of the best ways to observe these lines of force is to place a plate of glass over a magnet in contact with it, and then to sprinkle a quantity of very fine iron filings over the plate. On gently tapping the glass, the small particles, which act like so many compass needles under these circumstances, will arrange themselves under the influence of the magnetic forces into curves which coincide with the lines of force. If it is desired to preserve copies of such curves, the glass plate may be replaced by a sheet of white paper coated over with a very thin layer of paraffine. If this paper be then heated without disturbing the filings, these will sink into the melted paraffine, and on its cooling will remain embedded in it. Copies of this may then be taken by the ordinary photographic processes.

A better method is to use blue printing paper in place of that coated with paraffine, care being taken while exposing it to the sunlight not to disturb the filings. A still better method is to use an ordinary sensitized photographic plate, and, as with the blue paper, to perform the experiment in a darkened

room. The curves formed by the particles may be impressed on the plate by illuminating it with an electric spark, or by quickly turning on and off the light from an incandescent lamp. From a plate prepared in this way any number of prints can be readily taken.

A diagram of the lines of force taken by this method is exhibited in Fig. 1, in which case they are due to the combination of two dissimilar poles. The student will find it exceedingly interesting and instructive to make for himself a set of plates showing the lines of force produced by (1) an ordinary bar magnet, (2) a horseshoe magnet, (3) various irregular combinations of magnetic poles, (4) the pieces of a broken bar magnet placed close to each other, and (5) the presence of a piece of soft iron near the poles of a magnet.

From an inspection of Fig. 1 it can readily be seen that the lines of force are closer together near the poles of the magnet than they are at a distance from it. As the force exerted by the magnet on a unit pole decreases with the distance of the pole from the magnet, we may consider the number of lines of force perpendicularly crossing a given small area, taken at any point in the field, to be proportional to the force intensity at that point. In a field of uniform intensity, therefore, the lines of force will be so distributed that the same number will cut any such area placed as indicated.

II. DETERMINATION OF MAGNETIC MOMENTS.

From an inspection of its lines of force it is evident that a bar magnet cannot be considered as having a single pair of poles or centers of attraction. In fact, such a magnet possesses an indefinite number of pairs of poles, those of each pair being of equal strength and of opposite sign.

If, then, a bar magnet be suspended in a uniform horizontal magnetic field, similar to that represented in Fig. 2, where P denotes the direction towards which a north pole tends to

move, it will be acted on by a number of pairs of forces, whose resultant effect is that of a couple, and under the action

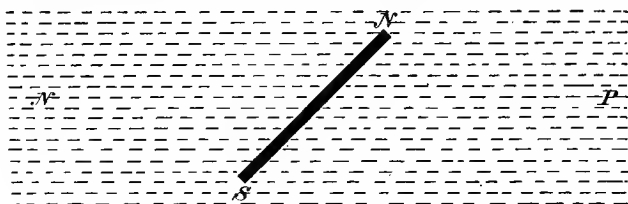


Fig. 2.

of this couple the magnet will tend to take up a position such that its axis coincides with the direction of the lines of force of the field.

If we represent the masses of the various pairs of poles by $\mu_1, -\mu_1; \mu_2, -\mu_2$; etc., and the distances between those forming each pair by l_1, l_2 , etc., and if H is the intensity of the magnetic field, the turning couple or moment of the forces about the point of suspension is equal to $H \cdot \sum \mu l \cdot \sin \theta$, the angle θ being that which the magnet makes in any position with the direction of the lines of force.

The expression $\sum \mu l$, which is generally written M , is a constant for each magnet, irrespective of how it may be situated, and is called its *magnetic moment*.

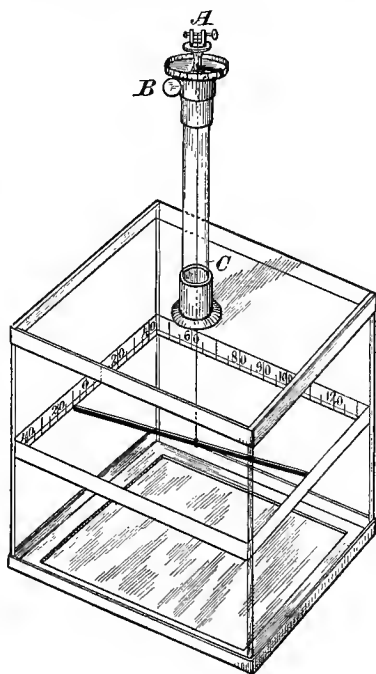


Fig. 3.

One of the best methods for comparing the magnetic moments

of different magnets is that of the torsion balance, and in case absolute determinations are required these can be made with a fair degree of accuracy by means of the same instrument.

Such a balance as ordinarily used (Fig. 3) consists of a glass case provided with a scale placed round its sides, and graduated so as to indicate degrees. Situated in the same plane as the scale is the magnet to be tested. It is suspended by a fine wire which, after passing through a glass tube, *C*, attached perpendicularly to the cover of the case, has its upper end fastened to a brass torsion head, *A*. This piece rests on a graduated brass collar, *B*, surrounding the upper portion of *C*, and by means of these graduations, and a mark on the torsion head, any angle through which the latter may be turned can be directly ascertained.

Having placed the balance in some suitable position, it is first necessary to find what two graduations on the scale lie in the earth's magnetic meridian. This is best done by placing a compass needle, resting easily on a pivot, at the center of the case, or by suspending it from *A* by a torsionless fiber, such as a single strand of silk. The needle will, under these conditions, lie in the meridian, and the graduations on the scale towards which it points may then be easily noted.

A bar of brass or copper of about the same dimensions as the magnet to be examined, should then be suspended by the wire, and the torsion head rotated until this bar rests in the meridian. If the bar of brass or copper be then detached without disturbing the wire, and the magnet put in its place, the latter will, after oscillating for some time, finally come to rest in the meridian without being subjected to any torsional strain. If then *A* be turned through some angle α , the magnet will be displaced from the meridian by an angle θ , and $(\alpha - \theta)$, which is generally denoted by ϕ , will be the amount of torsion in the wire.

When the magnet is in this position the torsion couple is balanced by the action of the forces due to the earth's magnetic

field, and since this may be taken to be uniform, we have, by taking moments about the point of suspension, the relation

$$HM \sin \theta = C\phi,$$

or,
$$M = \frac{C}{H} \cdot \frac{\phi}{\sin \theta};$$

H being the horizontal intensity of the earth's field, M the magnetic moment, and C the constant of torsion for the wire. (See Appendix B for method of determining C .)

A method of finding H is given in Experiment V., and as both this quantity and C can be ascertained by preliminary investigations, it only remains to determine an accurate value for $\frac{\phi}{\sin \theta}$.

This is done simply by repeating the experiment just described, a number of times, care being taken at each trial to give to A a different amount of rotation.

If it is desired to make a comparison of two magnets, it is only necessary to determine a value for $\frac{\phi}{\sin \theta}$ for each of them, provided the same torsion wire is used and both experiments are conducted in the same place. The ratio of these two quantities will be equal to that of the two magnetic moments, and if one of them is known, the other can then be found in terms of it.

The method is especially suitable for magnets which are long and slender. In case they are very short the angular deflections can be measured with much greater accuracy by adopting the method exhibited in Fig. 13.

In this experiment the best results are obtained by using fine silver wire or quartz fibers for the suspensions.

III. THE DECLINATION COMPASS.

It has already been pointed out that if a bar magnet be suspended so as to rotate freely in a horizontal plane, it will take up a position with its axis pointing in a northerly direction.

In Experiment I., it has also been shewn that the field of a magnet can be explored by a small compass needle being placed at different points in it, since the needle will come to rest in each position with its axis always coinciding with the direction of the lines of force.

Such considerations as these lead us to the conclusion that the earth is an immense magnet surrounded with a field of

force, and that therefore the bar magnet suspended as above comes to rest pointing towards the north simply because it performs the same function with regard to the earth's field that the small compass needle does with regard to that of a given magnet.

In order to obtain exact knowledge of the earth's magnetic action at a point, it is necessary to determine (1) the direction of the lines of force and (2) the intensity of the magnetic action at that point.

The directions of terrestrial lines of force are

defined with respect to the geographical meridian, and to the horizontal plane through any point. The vertical plane passing through the axis of a freely-suspended horizontal magnet at rest is called the *magnetic meridian*, and the angle between it and the geographical meridian, the *declination*.

Since the magnetic poles of the earth lie in the magnetic meridian, the lines of force at any place will be parallel to this plane, and, therefore, the first step to take in ascertaining their

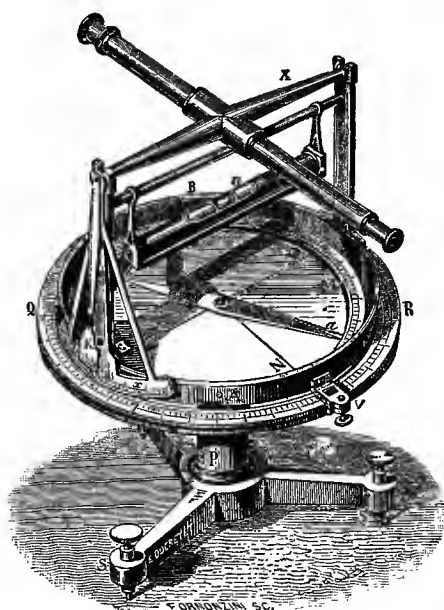


Fig. 4.

direction, is to make an accurate determination of the declination.

This is found by means of an instrument called the *declination compass*. It consists, as represented in Fig. 4, of a circular brass or copper box AB , attached at right angles to, and freely rotating about, a foot P , which is itself supported by a tripod. On the bottom of this box there is placed a divided circle M , at whose center a lozenge-shaped compass needle is delicately pivoted. To the box there are fastened two uprights supporting a horizontal axis X , to which is fixed an astronomical telescope, movable in a vertical plane. The foot P also carries a fixed azimuthal circle QR , and by means of it and the vernier V any rotation given to the box may be determined. The graduated arc α , attached to the upright and the vernier K , which moves with the telescope, afford a means of measuring any angle through which the latter may be turned.

In determining the declination, the divided scale in the bottom of the box is first made horizontal by using the level suspended from the telescope's support as an indicator, and the screws in the tripod for making the adjustment. If the geographical meridian through the point of observation has been previously determined, the telescope is then sighted upon some distant mark that also lies in it.

Since the divided scale is so placed that its zero diameter is in the same vertical plane as the axis of the telescope, it will also lie in the geographical meridian when this is done, and the angle between it and the axis of the compass needle in its position of rest will be the declination.

In case the geographical meridian has not been located, it will be necessary to sight the telescope upon the sun at noon, or on some star whose time of transit is given in the Nautical Almanac. When the telescope is so adjusted, the diameter of the azimuthal circle M lying in the meridian can then be ascertained, and the declination will as before be the angle between the axis of the compass needle and this diameter. As it frequently happens

that owing to imperfect magnetization the magnetic axis does not coincide with the geometrical axis of the needle, it should be inverted so that what was its lower face in the first case will be the upper in the second. The mean of the two readings obtained should be taken for the declination, and the whole operation repeated several times to insure accuracy.

The declination is said to be east or west according as the north pole of the needle is to the east or west of the geographical meridian. The results obtained from a series of observations, extending over a long term of years, indicate a gradual change at each point, the declination being at one time east or west, and after increasing up to a certain limit, then diminishing and passing to the other side of the meridian. Besides this change, which is very slow, there are yearly and daily variations, and, in addition to these, abrupt changes frequently occur owing to the existence of so-called magnetic storms.

Maps are issued each year from various magnetic observatories, shewing the declination at various places, and on these are drawn lines, called *isogonic*, which pass through those points where the declination is the same.

IV. THE INCLINATION COMPASS.

Having determined the magnetic meridian, or plane, parallel to the lines of force, it then remains to find the angle their direction makes with the horizontal. This angle is called the *inclination* or *dip*, and is determined by the inclination compass or dipping needle.

A common form of the instrument is exhibited in Fig. 5, where *m* is an azimuthal circle supported on three feet supplied with leveling screws, and *A* is a plate which carries a spirit level, and is free to rotate about a vertical axis. A frame *r*, resting on two uprights, supports the graduated circle *M* in a vertical plane, and *ab*, a magnetic compass needle, rotates about a horizontal axis through its center of gravity, the resting

points being pieces of agate embedded in the frame at the center of the vertical circle. The instrument is so constructed that when the plate *A* is level, the zero diameter of the circle *M* is horizontal.

In determining the inclination, the instrument must first be leveled and then any of the following methods may be applied :

METHOD I. — Turn the frame *A* very slowly about its vertical axis of rotation, and observe the values of the inclination during the motion. The greatest dip noted will be the true inclination at the point of observation, as it is obtained when the vertical circle is in the magnetic meridian, and the needle is then acted upon by the total resultant of the earth's magnetic action.

METHOD II. — Rotate the plate *A* until the needle assumes a vertical position and points towards the ninety-marks on the circle *M*. Since the needle is suspended at its center of gravity, and in this position is acted upon only by the vertical component of the terrestrial field, the circle *M* must then be at right angles to the magnetic meridian. By next turning it through a right angle, as indicated by the azimuthal circle *m*, the needle will be in the meridian, and the inclination may be noted.

METHOD III. — By this method the vertical circle is placed successively in two planes which are at right angles to each other, and are situated one on each side of the meridian. The angle between the needle and the horizontal diameter is noted

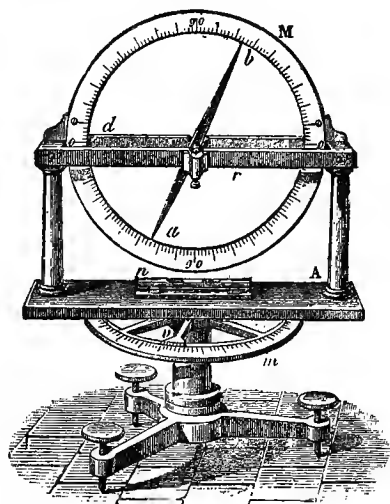


Fig. 5.

in each case, and from these angles the inclination at the point of observation is determined.

The disposition is that shewn in Fig. 6. AHB represents the magnetic meridian, AEB a plane making an angle θ with it, and ACB a plane at right angles to AEB . D_1 and D_2

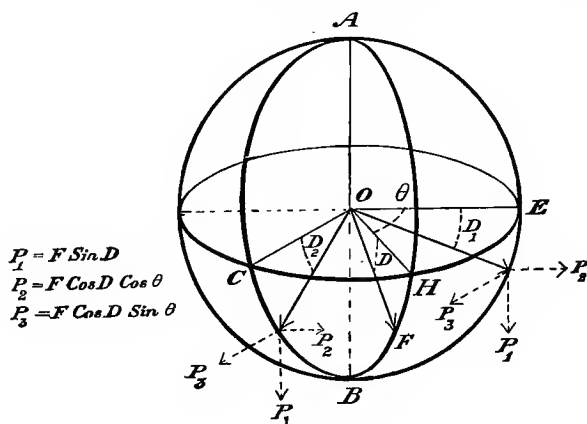


Fig. 6.

denote the angles made by the needle with the zero diameter of the vertical circle when it is at rest in each of these positions.

If D is the true value of the inclination at points in the immediate neighbourhood of the observing station, and F is the resultant intensity of the earth's magnetic action on the needle there, the latter can be resolved into two components in the magnetic meridian: one horizontal $F \cos D$, and the other vertical $F \sin D$. The component $F \cos D$ can again be resolved in an infinite number of ways into two others in the horizontal plane. In a direction making an angle θ with the meridian, it has a component $F \cos D \cos \theta$, and in one perpendicular to this, a component $F \cos D \sin \theta$. These forces are represented in the figure as acting on the needle when it is at rest in the plane AEB .

As the force $F \cos D \sin \theta$ acts perpendicularly to this plane, the position occupied by the needle is not affected by it, and

the angle D_1 is therefore completely determined by the relation

$$\cot D_1 = \frac{F \cos D \cos \theta}{F \sin D},$$

$$\text{or} \quad \cot D_1 = \cot D \cos \theta. \quad (1)$$

From similar considerations it can be readily seen that when the vertical circle is turned into the plane ACB the component $F \cos D \cos \theta$ does not affect the displacement of the needle, and the angle D_2 is therefore given by

$$\cot D_2 = \cot D \sin \theta. \quad (2)$$

Eliminating θ from (1) and (2), it follows that

$$\cot^2 D = \cot^2 D_1 + \cot^2 D_2.$$

If, therefore, the *apparent* inclinations D_1 and D_2 are found in any two planes at right angles to each other, the *true* inclination can be calculated by applying this relation.

METHOD IV. — If when the divided circle M is in the magnetic meridian the needle be given a slight displacement from its position of rest, it will oscillate harmonically, and if F is the intensity of the earth's field, M_1 the magnetic moment of the needle, and M_2 its moment of inertia about the axis of rotation, we have the time of a small oscillation given by

$$t_1 = 2\pi \sqrt{\frac{M_2}{M_1 F}},$$

since the equation of motion is

$$M_2 \frac{d^2\theta}{dt^2} = -M_1 F \sin \theta.$$

If the circle be now turned so that its plane is at right angles to the magnetic meridian, and the needle be again made to perform small oscillations, its periodic time is given by $t_2 = 2\pi \sqrt{\frac{M_2}{M_1 F \sin D}}$, since in this case the vertical component of the earth's attraction alone is acting.

We have, therefore, $\sin D = \frac{t_1^2}{t_2^2}$ and from this equation the inclination may be calculated. The times t_1 and t_2 should be found by allowing the needle to oscillate for a considerable time.

Method III. will probably give the most satisfactory results, but the sources of error are so numerous in any case that the greatest care must be taken in making the adjustments if accurate determinations are required.

Precautions.—I. It frequently happens that from various causes the vertical circle becomes displaced so that the zero diameter is not horizontal when the plate *A* is level. It then becomes necessary to ascertain the true horizontal diameter.

Since the needle will be vertical when its plane of rotation is at right angles to the magnetic meridian, it will also be vertical when the circle *M* is turned through an angle of 180° from this position. Two points on this circle must therefore be found such that the needle points towards them when it is in one position, and also when the same circle is given a rotation of 180° . This will determine at the same time both the magnetic meridian and the two points in the vertical diameter of the graduated circle. The horizontal diameter can then be inferred.

II. Readings should be taken at both ends of the needle in order to correct any error arising from the axis of rotation not passing exactly through the center of the vertical circle.

III. The magnetic axis of the needle may not coincide with its geometrical axis, and to correct errors from this source, the method of reversion indicated in Experiment III. should be adopted. Instead of removing the needle, this can be accomplished by simply rotating the vertical circle through 180° when any reading has been taken, and again observing the inclination.

IV. The center of inertia may not lie in the axis of suspension. In this case, the inclination will be affected by gravity, and

should be corrected by repeating the operation after the needle has been remagnetized so that its poles are reversed. As in the case of *declination*, maps are used shewing the *inclination* at various places. The lines which pass through those points where the inclination is the same are termed *isoclinic*.

V. DETERMINATION OF THE ABSOLUTE INTENSITY OF THE EARTH'S MAGNETIC FIELD.

In Experiment IV. it has been shewn that if the dipping needle be slightly displaced from its position of equilibrium in the magnetic meridian, it will perform small oscillations whose periodic time is $2\pi\sqrt{\frac{M_1}{MF}}$, M_1 being the moment of inertia of the needle about the axis of rotation, M its magnetic moment, and F the resultant intensity of the earth's magnetic action. It follows from this that $MF = \frac{4\pi^2 M_1}{t^2}$, and since M_1 and t can be easily ascertained, a relation can thus be established between M and F . Owing to friction, however, an accurate result cannot be determined in this way, and it is customary to determine instead the product MH , H being the horizontal component of F , and then by means of a second relation $\frac{M}{H}$, to evaluate both the quantities H and M .

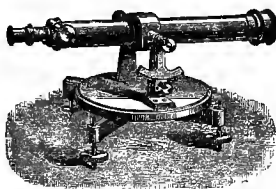


Fig. 7.

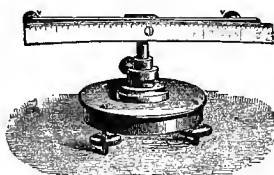


Fig. 8.

After H has been found in this manner, and the *inclination* determined at the point of observation by a separate investigation, the resultant intensity F can then be calculated from the relation $F = H \cdot \sec D$.

The apparatus employed in this experiment consists of a telescope (Fig. 7) mounted so as to be capable of rotation in a horizontal and a vertical plane, a finely divided scale VV (Fig. 8), and a magnetometer.

This instrument (Fig. 9) is composed essentially of a cylindrical brass or copper box B resting on a tripod provided with leveling screws. It is capable of rotation about a vertical axis, carries with it a graduated circle C attached to its base, and has its anterior face pierced by a circular opening in which is inserted a convergent lens O whose focal length is about one meter. A metallic column V has its base inserted in a beveled plate resting on the top of B , and carries at its upper end a reel on which is wound the silk strand which supports the stirrup and the bar magnet A . This stirrup carries a vertical mirror M mounted so as to be at right angles to the magnet when the latter is in position. A second mirror M' is fitted into a frame fastened to the bottom of the box, and can be adjusted either horizontally or vertically by means of the screws E, E . A divided circle C' indicates any rotation given to the column V .

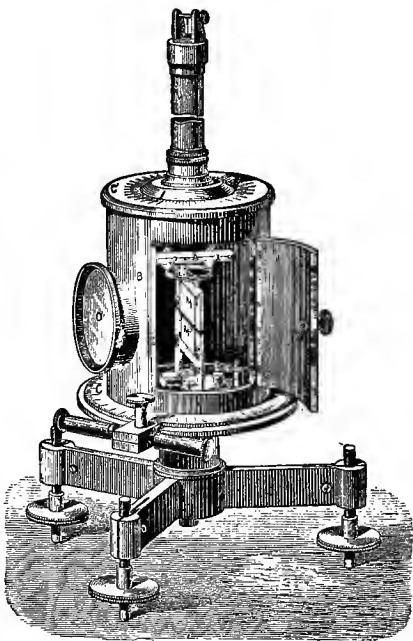


Fig. 9.

Theory. — *Determination of MH .* When a bar magnet is suspended so that it is free to move in a horizontal plane, it will,

when displaced, oscillate about its mean position. If H is the horizontal intensity of the terrestrial field, and M the magnetic moment of the bar, the equation of motion is

$$\Sigma mr^2 \frac{d^2\theta}{dt^2} = -MH \sin \theta. \quad (1)$$

The time of a small oscillation is therefore given by

$$t = 2\pi \sqrt{\frac{\Sigma mr^2}{MH}},$$

and denoting the moment of inertia of the magnet by M_1 , this equation gives the relation

$$MH = \frac{4\pi^2}{t^2} M_1. \quad (2)$$

The quantity t can easily be ascertained by counting the oscillations performed in some given time, and M_1 can either be calculated from the dimensions of the magnet or found experimentally.

In the case of very exact determinations allowance will have to be made for the torsion couple of the suspending fiber. Adopting this correction, the relation becomes

$$MH = \frac{4\pi^2 M_1}{t^2} - C, \quad (3)$$

the constant C being the coefficient of torsion, which must be ascertained by a preliminary investigation. (See Appendix B.)

Determination of $\frac{M}{H}$.—The relation $\frac{M}{H}$ is determined from the reciprocal actions exerted by two magnets situated at a distance from each other, which is great compared with their lengths. After having accurately determined the time of vibration of the magnet A , it is next used to deflect a second one which is inserted in the stirrup in its place.

Either of two methods may be followed. In one the disposition is that shewn in Fig. 10, where NS represents the magnet

in the stirrup at rest in the meridian, and AB is the magnet A resting on a support in the same horizontal plane as NS , but pointing east and west. In the second, the arrangement is

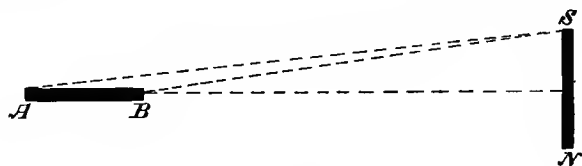


Fig. 10.

shewn in Fig. 12. The magnet NS is suspended in the meridian as before, but the deflecting magnet AB is now placed with its center in this plane and its axis at right angles to it.

METHOD I. — Denoting the lengths of AB and NS (Fig. 10) by l and l_1 , respectively, and the strengths of their poles by

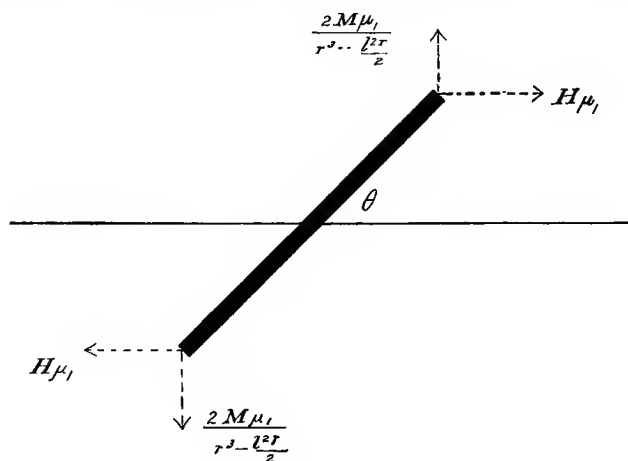


Fig. 11.

μ and μ_1 , it follows if r , the distance between their centers, is considerable, that the action exerted by AB on each of the poles of NS is approximately equal to

$$\frac{2 M \mu_1}{r^3 - \frac{l^2 r}{2}}$$

and may be taken as acting parallel to the line joining the centers of the magnets. Under these actions and those exerted by the earth's field, the magnet NS will take up the position indicated in Fig. 11. Taking moments about the point of suspension, it follows that

$$\frac{2\mu_1 l_1 M \cos \theta}{r^3 - \frac{l^2}{2}r} = H\mu_1 l_1 \sin \theta,$$

from which
$$r^3 \tan \theta = 2 \frac{M}{H} \left(1 + \frac{l^2}{2r^2} \right). \quad (4)$$

If now this experiment be repeated with the centers of the magnets at a distance r_1 , apart, we will have a second relation,

$$r_1^3 \tan \theta_1 = \frac{2M}{H} \left(1 + \frac{l^2}{2r_1^2} \right), \quad (5)$$

and $\frac{M}{H}$ is therefore given by the equation,

$$\frac{M}{H} = \frac{1}{2} \frac{r^5 \tan \theta - r_1^5 \tan \theta_1}{r^2 - r_1^2}. \quad (6)$$

METHOD II.—Denoting again the distance between the centers of the magnets by r , and their lengths by l and l_1 , the

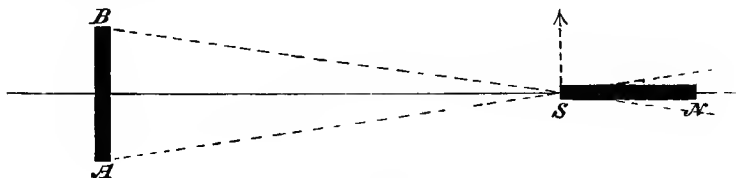


Fig. 12.

actions exerted by AB on S and N respectively (Fig. 12) are perpendicular to the meridian, and are approximately equal to

$$\frac{\mu_1 M}{\left(r - \frac{l}{2}\right)^3} \quad \text{and} \quad \frac{\mu_1 M}{\left(r + \frac{l}{2}\right)^3}$$

Under these forces, and those exerted by the earth's field, the suspended magnet will again be deflected from the meridian (neglecting any small motion of translation) through an angle θ , the equation of moments being, in this case,

$$\frac{\mu_1 l_1 M \cos \theta}{2} \left\{ \frac{1}{\left(r - \frac{l_1}{2}\right)^3} + \frac{1}{\left(r + \frac{l_1}{2}\right)^3} \right\} = \mu_1 l_1 H \sin \theta. \quad (7)$$

From this it follows that

$$r^3 \tan \theta = \frac{M}{H} \left\{ 1 + \frac{3}{2} \frac{l_1^2}{r^2} \right\}, \quad (8)$$

and repeating the experiment, taking the distance between the centers to be r_1 , we have also

$$r_1^3 \tan \theta_1 = \frac{M}{H} \left\{ 1 + \frac{3}{2} \frac{l_1^2}{r_1^2} \right\}. \quad (9)$$

Combining (8) and (9), a second equation for determining $\frac{M}{H}$ is given by

$$\frac{M}{H} = \frac{r^5 \tan \theta - r_1^5 \tan \theta_1}{r^2 - r_1^2}. \quad (10)$$

Experiment. — When the apparatus is arranged for taking observations, the several parts are situated as in Fig. 13, where AB represents the meridian, CA the telescope, VV' the scale, and GH and KL the mirrors attached to the base of the magnetometer and to the stirrup respectively. The first step to take in making this disposition is to adjust the mirror GH , so that it is perpendicular to the meridian. This is arrived at by placing a copper bar in the stirrup, and, after taking all the torsion out of the silk strand, by turning the column V until the bar lies approximately in the meridian, as indicated by a delicately poised compass needle. On replacing the copper bar by the magnet, the latter will then come to rest in the meridian without being subjected to any torsion, and the mirror KL will therefore be perpendicular to it. The magnetometer is next turned until the mirror GH is parallel to KL . This can be

performed very accurately by observing with the telescope the images of the scale (GH being slightly inclined to the vertical) formed by the two mirrors, the latter being parallel when the divisions on one scale image are seen directly above the corresponding ones on the other.

In order to adjust the telescope so that its axis will be in the meridian, place the scale with its stand on the same support as the former, and then, after noting the scale division,

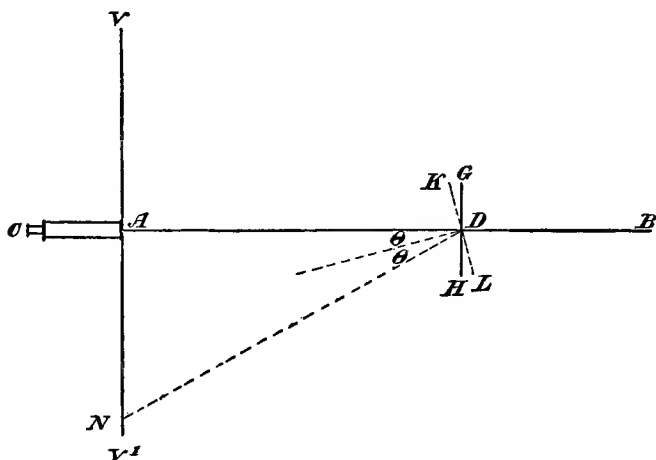


Fig. 13.

generally zero, which is situated in the same plane as the axis of the telescope and the vertical diameter of its objective, move the support until the images of this division, formed by the mirrors, are seen on the vertical cross-hair in the eyepiece. The axis of the telescope will then be in the meridian, and the scale at right angles to it. In determining the quantity MH , it is first necessary to find accurately the time of a single oscillation. This is done by giving the magnet A initially a small displacement, by bringing near to it a piece of iron or steel, and when the motion has become steady, by ascertaining with a stop watch the time occupied in performing a large

number (one hundred, for example) of small oscillations. The time of a single one can then be calculated, and by taking the mean of the results obtained by repeating this operation a number of times a very close approximation can be found. In order to make certain that the stop watch marks correct time it should be compared, both before and after the experiment, with a standard chronometer.

As already indicated, the moment of inertia of the magnet A and the stirrup upon which it rests may be determined experimentally by the method indicated in Appendix B; but for a rough approximation that of the stirrup may be neglected, and that of the magnet calculated from its dimensions.

In determining $\frac{M}{H}$, a second magnet is placed in the stirrup, and the magnet A rests on a support attached to a copper bar (not shewn in the figure), which together with a counterpoise rotates about the cylindrical column connecting the box of the magnetometer to the tripod. Divisions are marked on this bar, shewing the distance of the center of the support from the axis of rotation of the instrument, and therefore that between the centers of the magnets. The mirror GH is generally attached to the base of the magnetometer so that its lower edge lies in the ninety-diameter of the divided circle C , and therefore when the bar carrying A is turned so that its index points to this division it is perpendicular to the meridian, and $\frac{M}{H}$ may then be found by Method I.

If it is desired to adopt Method II., the bar should be turned through a right angle from this position.

The method of Poggendorff is followed in measuring the angle through which the auxiliary magnet is deflected. When this magnet is in the meridian, one sees the image of the zero division (Fig. 13) formed on the cross-hair of the telescope, but when the mirror KL is turned through an angle θ , one sees formed there the image of some other division, N .

It follows then that

$$\tan 2\theta = \frac{AN}{AD},$$

or
$$\theta = \frac{1}{2} \cdot \frac{AN}{AD} \text{ approximately.}$$

In ascertaining the value of θ corresponding to a given value of r the magnet A should first be placed with its positive pole

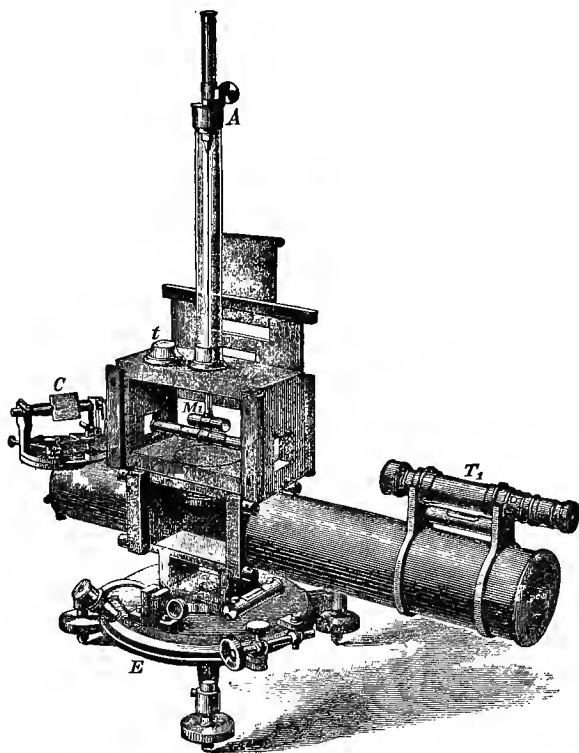


Fig. 14.

pointing east, for example, and the deflection noted, and it should then be turned with its negative pole in this direction, and the deflection again observed. After this the bar should

be rotated through 180° and the same process repeated, the mean of the four values so obtained being the one inserted in the formula with the corresponding value of r .

The apparatus is designed specially for the determination of the quantity H , and is not suited for finding the magnetic moments of magnets of different sizes.

When, however, H has been determined accurately for the point of observation, these can readily be obtained with a slightly modified form of the instrument, by simply performing the first operation in this experiment.

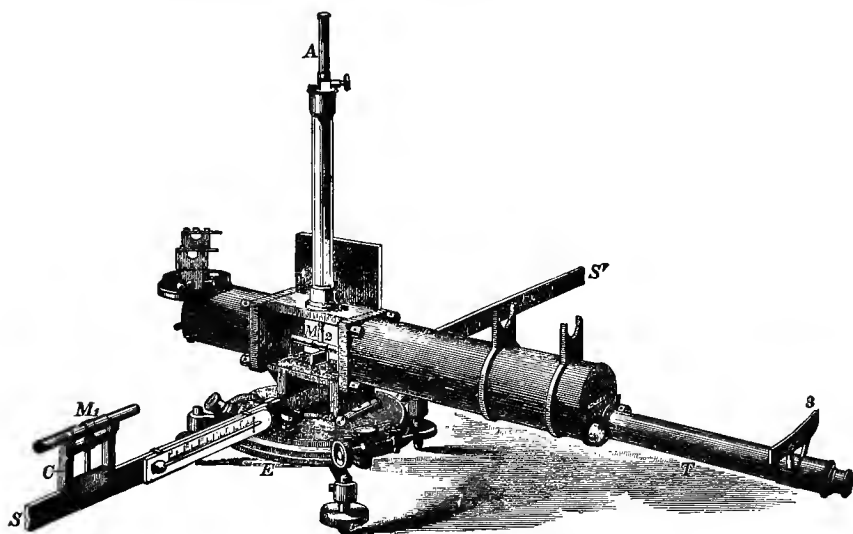


Fig. 15.

In the British magnetic observatories, and also, in fact, in many laboratories, the Kew portable magnetometer is used for finding the horizontal intensity.

In Fig. 14 it is shewn adjusted for determining the quantity MH . The vibration magnet M_1 is suspended at the center of a wooden box by a silk thread attached to the torsion head A . The magnet is a hollow steel cylinder having a glass scale at

one end and a lens at the other, the former being in the focal plane of the lens. This scale is illuminated by a small mirror, C , and the oscillations of the magnet are observed by means of the telescope T_1 .

The moment of inertia of the magnet is found experimentally, by inserting a cylindrical brass or copper bar whose moment of inertia can be calculated, in the brass collar which is shewn immediately above the magnet.

The arrangement adopted in determining $\frac{M}{H}$ is exhibited in Fig. 15. The wooden box is removed and a metallic one put in its place. The vibration magnet M_1 is placed on the support C , which is movable along the bar SS' previously referred to, and the telescope T , with its accompanying scale s , is attached to the lower part of the end of the large hollow cylinder. The auxiliary magnet, M_2 , and the mirror for reflecting the scale divisions are shewn in position in the figure.

In determining H by this instrument the same theory and the same general method of manipulation as that just described apply.

VI. MAGNETIC FIELD OF A CURRENT.—BIOT AND SAVART'S LAW.

Oerstedt discovered in 1820 that a magnetic needle, if suspended in the neighbourhood of a rectilinear current, tended to place itself at right angles to the conducting wire. This discovery at once suggested that surrounding a rectilinear current there is a magnetic field and that its lines of force are a series of concentric circles having the conductor as a common axis. The truth of this conclusion has been amply verified by experiment and can be clearly exhibited by the use of iron filings.

If a straight wire in which a current is established be passed up through a piece of cardboard which is held horizontally, and fine iron filings be then sprinkled upon it, these will at once arrange themselves into the series of circles described ; while, on

the other hand, if the wire be laid flat on the paper, the filings will arrange themselves into a series of straight lines cutting the line of the current at right angles.

Since then the lines of force are circles, it only remains to state the sense in which a north magnetic pole tends to move, in order to define completely the action exerted by a given current on a magnet. The following rule has been proposed by Ampere for guidance in this connection: *The positive pole of a magnet tends to turn towards the left hand of an observer looking at it, and supposed swimming in the conductor in the direction in which it has been agreed to consider the current as passing.*

A. The magnetic field of an infinitely long rectilinear current.

As in the case of other magnetic fields the intensity of that produced by a long rectilinear current may be investigated by the method of oscillations. The current is passed through a

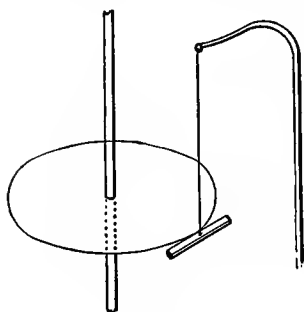


Fig. 16.

vertical conductor as in Fig. 16, and a small magnet is suspended by a torsionless fiber at different points along a horizontal line passing through the wire perpendicularly to the direction of the magnetic meridian at that point.

If the needle is taken very small and is placed not too close to the wire, the action of the current on it will reduce to that of a couple which

acts in conjunction with, or in opposition to, the horizontal component of the earth's magnetic field according to the direction in which the current passes.

By applying this method Biot and Savart found that when a constant current was maintained in the wire the intensity of the field was *inversely proportional to the distance* from the conductor.

Denoting the horizontal component of the earth's action near the current by H , and the intensity of the field due to the cur-

rent at the distances a and b by F_1 and F_2 respectively, the equations of motion for the magnet when acted on by (1) H alone, (2) F_1 and H together, and (3) F_2 and H together, are

$$\Sigma m r^2 \frac{d^2 \theta}{dt^2} = -HM \sin \theta, \quad (1)$$

$$\Sigma m r^2 \frac{d^2 \theta}{dt^2} = -(H + F_1)M \sin \theta, \quad (2)$$

and
$$\Sigma m r^2 \frac{d^2 \theta}{dt^2} = -(H + F_2)M \sin \theta. \quad (3)$$

If n , n_1 , and n_2 are the number of oscillations performed in each of these cases respectively in a given time, we have, if the amplitudes of the vibrations are small,

$$n^2 = KH, \quad (4)$$

$$n_1^2 = K(H + F_1), \quad (5)$$

$$n_2^2 = K(H + F_2), \quad (6)$$

where K is a constant depending on the moment of inertia, and the magnetic moment of the magnet.

From (4), (5), and (6) it follows that

$$\frac{F_1}{F_2} = \frac{n_1^2 - n^2}{n_2^2 - n^2}. \quad (7)$$

If then the law of the inverse distance holds, we should have $\frac{F_1}{F_2} = \frac{b}{a}$, and therefore $\frac{b}{a} = \frac{n_1^2 - n^2}{n_2^2 - n^2}$, i.e. we should have $b(n_2^2 - n^2) = a(n_1^2 - n^2) = \text{a constant for all positions of the magnet along a line through the conductor perpendicular to the meridian.}$

The apparatus for performing this experiment is similar to that shewn in Fig. 9. The various parts are arranged just as indicated there excepting the wire conductor, which is movable about the vertical through the point of suspension of the magnet, and is so placed that the perpendicular on it from the magnet is at right angles to the meridian.

In order to make this adjustment accurately the wire is first placed approximately in position, and the magnet is brought to rest in the meridian. If the current is then turned on and the magnet remains still at rest the adjustment is perfect; if, however, there is a displacement of the magnet the wire must be rotated through a small angle in a sense opposite to that of such displacement and the same test again applied.

As the chief difficulty in this experiment arises from the inconstancy of the current, a sensitive galvanometer should be inserted in the circuit along with an adjustable resistance, so that any variation in the current strength may be at once detected, and corrected by altering the resistance. A battery of Grove cells will be found to produce a very steady current for a considerable time.

In order to correct still further any variations in the current, the oscillations of the magnet should be counted a number of times for each position.

B. Magnetic field produced by an angular current.

Biot and Savart modified the experiment just described by observing the action of a current traversing two very long rectilinear conductors BA and AC , so connected as to make an angle at A , Fig. 17, upon a short magnet placed with its center P on the line bisecting the angle BAC .

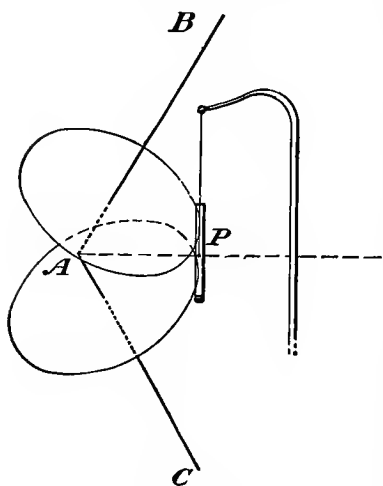


Fig. 17.

As can be seen from the figure, the circular lines of force due to the current passing along BA and AC have a common tangent at P , and if these wires are so placed that their plane is vertical and per-

pendicular to the magnetic meridian through A , the action exerted on a short magnet suspended horizontally at P will again be reduced to two equal and opposite forces whose direction is in the meridian at that point.

The results of experiment indicate that the action of such a disposition of conductors is given by

$$F = \frac{K \tan \frac{A}{2}}{r},$$

where A is the angle BAP , and r is the length AP , K being a constant.

The same method is adopted in verifying this law as in that of the inverse distance for an infinite rectilinear current, and similar precautions are necessary regarding the constancy of the current, and the plane BAC being perpendicular to the meridian.

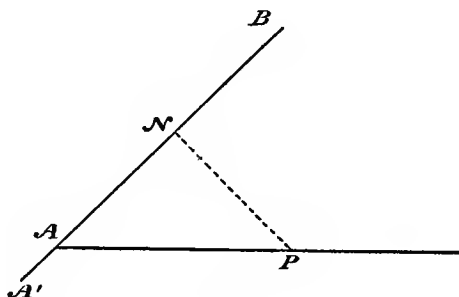


Fig. 18.

Law of Laplace.

It follows from the law just investigated that the action of an infinite current, supposed commencing at A (Fig. 18), upon a magnetic pole placed at P can be represented by

$$F = \frac{K}{2} \frac{\tan \frac{A}{2}}{r},$$

where BAP is the angle A , and AP is the length r .

It follows, therefore, that

$$F = K \frac{\sin^2 \frac{A}{2}}{p}$$

where $p = PN$, and since the action of AB on P may be considered as the sum of the elementary actions of its parts, that of an element AA' is given by

$$\begin{aligned} dF &= K \frac{\sin \frac{A}{2} \cos \frac{A}{2}}{p} dA \\ &= \frac{K \sin A dA}{2p}; \end{aligned}$$

i.e. since $\frac{dA}{AA'} = \frac{\sin A}{r},$

and $p = r \sin A,$

$$dF = \frac{K}{2} \frac{AA' \sin A}{r^2},$$

or, denoting the length AA' by ds , we have Laplace's law for the action of an element of current,

$$dF \propto \frac{ds \sin A}{r^2}.$$

VII. MEASUREMENT OF CURRENT INTENSITY.

Of the many effects that can be produced by an electric current, two have been selected as being most suitable for the determination of its intensity,—the action exerted by the current on a magnetic pole, and the decomposition of chemical compounds.

If a circuit be so arranged that a constant current in passing can produce (1) magnetic, (2) heating, (3) chemical, and (4) magnetizing effects, it will be found that if a measure of each of these be taken, only those of the magnetic and the chemical effects will bear the same ratio to the new values obtained when the current is by any means altered in intensity.

It has therefore been agreed to consider current strength as being proportional to the action in each of these cases; and *vice versa*, the action exerted on a magnetic pole, and the amount of chemical separation produced, will therefore be proportional to the intensity of the current causing such action.

If, then, a magnetic pole of strength μ be placed near a conductor bearing a current whose intensity is C , Laplace's law, when combined with the above convention, gives

$$dF = K\mu \frac{Cds \sin A}{r^2},$$

as the force exerted on the pole by an element of current ds , A being the angle between the line r and the element ds .

The absolute unit of current strength in the electro-magnetic system is defined to be such that if one centimeter length of the circuit be bent into an arc of one centimeter radius, the current in it exerts a force of unit intensity upon a magnetic pole of unit strength placed at the center of the arc. On the basis of this definition, therefore, $\mu \frac{Cds \sin A}{r^2}$ will represent the force in absolute units exerted on the magnetic pole μ placed as previously indicated.

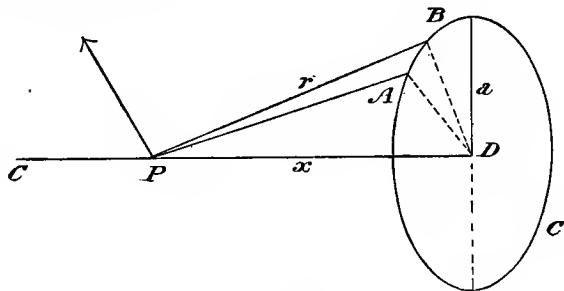


Fig. 19.

Action of a circular current on a magnetic pole.

Let AC represent the circular conductor bearing the current whose intensity is C (Fig. 19), AB or ds an element of this.

current, and P the position of a magnetic pole of strength μ , supposed placed on CD , the axis of the circuit.

Since in this case the angle A is a right angle, the force exerted by the element ds on the pole μ is equal to $\frac{\mu C ds}{r^2}$. It also acts in a direction at right angles to APB . If a is the radius of the circle AC , and PD be denoted by x , the component of this force parallel to CD is equal to $\frac{\mu \cdot C \cdot ds \cdot a}{(a^2 + x^2)^{\frac{3}{2}}}$. Since a similar expression may be obtained for each element of the circuit, the action of the whole conductor resolved along CD is given by

$$F = \frac{2 \pi a^2 \mu C}{(a^2 + x^2)^{\frac{3}{2}}}, \quad (1)$$

and from the symmetry of the arrangement it is evident, moreover, that this is the resultant action of the current.

If instead of a single coil there are n of them, the action will be given by

$$F = \frac{2 \pi n a^2 \mu C}{(a^2 + x^2)^{\frac{3}{2}}}, \quad (2)$$

and if the pole is situated at the center of the coil, this further reduces to

$$F = \frac{2 \pi \mu C n}{a}. \quad (3)$$

The tangent galvanometer.

A form of the tangent galvanometer commonly used is that shewn in Fig. 20. It consists of a coil of insulated wire wound on the vertical circle AB , which is capable of rotation about an axis coinciding with its vertical diameter. At the center of this circle a short magnet is suspended from C by a torsionless thread of silk, and immediately below it there is placed a horizontal divided circle DE , which can rotate about the same axis as AB , but independently of it. The divided circle FG , which is provided with a vernier, indicates the amount of any

rotation given to the coil bearing the current, and an aluminium pointer attached to the magnet serves to shew the angle through which the latter may be deflected.

In adjusting the instrument for a measurement, the circles *DE* and *FG* are first made horizontal by means of the leveling screws, and the coil *AB* is then turned until the magnet when at rest lies apparently in its plane, which will then coincide approximately with the magnetic meridian.

Since the magnet is short, the actions exerted on its poles by the current will form a couple which will tend to make it take up a position at right angles to the meridian. Owing, however, to the presence of the terrestrial field, it will come to rest in a position such as that indicated in

Fig. 21, where *AB* is the meridian, *CD* the magnet, and θ the angle of deviation.

When, therefore, the current is passing and the magnet has come to rest, this angle θ should be noted; and if, when the current is reversed, the magnet is deflected through the same angle but on the other side of the meridian, the plane of the coil coincides with the latter, and the instrument is ready for a measurement. If, however, this condition does not obtain, the coil *AB* must be given a slight displacement, and the test again applied.

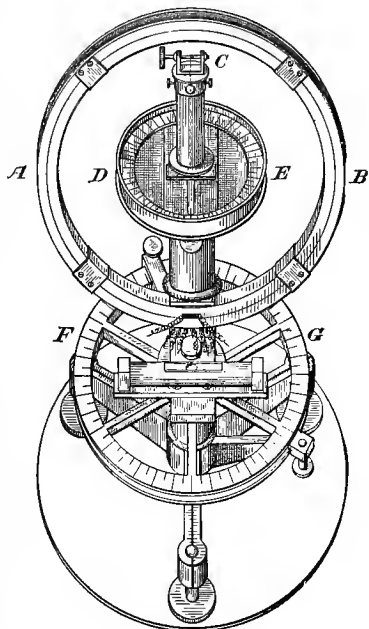


Fig. 20.

When the magnet is displaced, as in Fig. 21, it follows, by taking moments about the point of suspension that,

$$H\mu l \sin \theta = \frac{2\pi\mu l C n \cos \theta}{a},$$

or
$$C = \frac{Ha}{2n\pi} \tan \theta, \quad (4)$$

and since the right-hand member of this equation involves quantities which can be determined with precision, this method

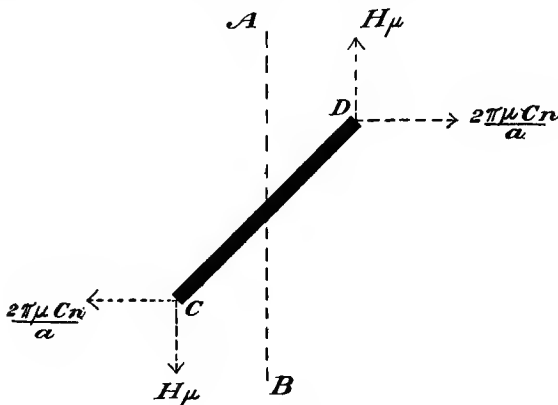


Fig. 21.

affords a means of accurately measuring the intensity of a current in absolute units.

The *ampere*, or practical unit of current strength, is equal to one-tenth of the absolute unit. The relation for determining the intensity of a given current in amperes is, therefore,

$$C = \frac{5}{n\pi} \frac{Ha}{\pi} \tan \theta. \quad (5)$$

To obtain satisfactory results, H must be found by means of the magnetometer at the point where the galvanometer is placed, and extreme care must be taken to have the coil AB accurately in the plane of the meridian. In order to overcome any imperfection in this adjustment, the current should always be reversed and the mean of the two readings taken.

For very close determinations, it is best to have only one turn of wire in the circuit, as in that case the radius a can then be measured very accurately.

In order to dampen the oscillations of the magnet, a plate of copper is frequently placed immediately below it within the circle DE ; and, as it generally takes a considerable time for the magnet to cease vibrating, even when this precaution is taken, its position of equilibrium is generally deduced by noting three of its successive positions of rest.

In many cases the angle of deflection is determined by means of a small mirror attached to the magnet. A scale and telescope may then be used just as in Experiment V., and the angle through which the needle is deflected calculated by the method there explained.

Besides being used for measuring given currents, the tangent galvanometer is frequently employed to test the accuracy of direct reading ammeters. These again, when they have been accurately calibrated, may be used to calculate the constant K in the formula $C = K \tan \theta$, for a tangent galvanometer whose construction is of such a character that it is difficult to measure with accuracy the dimensions of its coil.

Sine galvanometer.

The instrument just described may also be used as a sine galvanometer. The same precautions are to be taken in regard to placing the coil AB initially in the meridian, and the same adjustments made, except that when the current is turned on, and the magnet is deflected, the coil AB is then slowly and carefully rotated until it overtakes the magnet, which when it comes to rest does so with its axis in the plane of the coil.

Figure 22 exhibits the forces which then maintain the magnet in equilibrium, AB denoting the meridian. By again taking moments about the point of suspension, we obtain the relation,

$$C = \frac{Ha}{2n\pi} \sin \theta, \quad (6)$$

and from it the intensity of any current can be calculated. The angle θ , which is that through which the coil is rotated, can be ascertained by means of the graduations on FG . Especial care should be taken to have the pointer attached to the magnet directed in its initial and final positions to the same division on the graduated circle DE , which rotates with the coil.

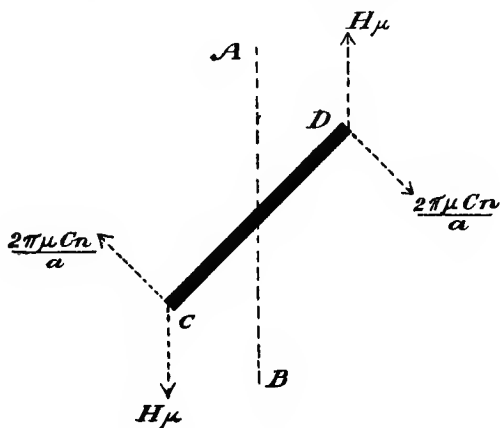


Fig. 22.

VIII. HYDROGEN VOLTAMETER.

When a current of electricity is made to decompose an electrolyte it is found that the amount of a gas liberated, or the quantity of a metal deposited, is directly proportional to the intensity of the current, and, within very wide limits, is not affected by the size or shape of the vessel containing the electrolyte.

The mass of a substance liberated in one second by one ampere of current is termed its *electro-chemical equivalent*. When, therefore, a constant current is passed through a voltameter and a known weight of gas is liberated in a second, the strength of this current is perfectly defined, and can readily be calculated when the electro-chemical equivalent of the gas is known. That of hydrogen is .00001038 grams.

A simple form of hydrogen voltameter is shewn in Fig. 23. The acidulated water is placed in the vase *AB*, whose base is pierced by two holes in which are cemented two small strips of platinum. These are connected to the two binding poles *C* and *D*, at which the current is made to enter and leave the

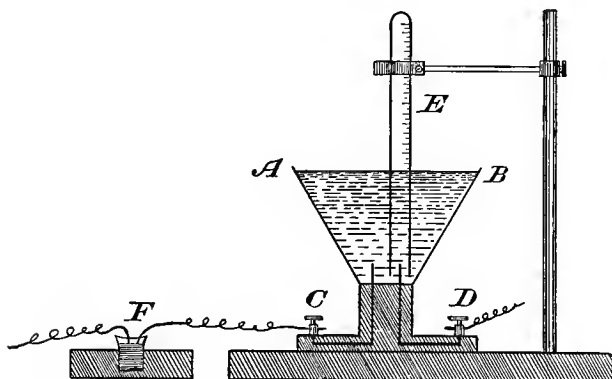


Fig. 23.

voltameter. The hydrogen is collected in the graduated glass tube *E*, and the current is made to pass through the mercury in the small cup *F*, so that the circuit can be rapidly made or broken.

When the current is established, the oxygen which appears at the positive platinum partly recombines with the water, and since the quantity of it which might be collected would not then be an exact measure of the decomposition which has taken place, it is customary to allow it to escape and to collect only the hydrogen, by placing the tube *E* over the negative platinum.

When an experiment is about to be performed the circuit is first broken at *F*, and the tube *E* is then carefully filled with the acidulated water and inverted over the negative electrode. A thermometer is also placed in the liquid in the vase.

The circuit is then closed by inserting the leading wire in the mercury, and a stop watch, reading to one-fifth of a second,

is started at the same instant. When the tube has been filled with hydrogen until the liquid in it stands at the same level as that in the vase outside, the circuit is broken and the watch stopped. The time, t , during which the current was passing can thus be accurately found, and the volume, V , in cubic centimeters, occupied by the hydrogen read directly from the graduations on the tube. The temperature of the gas, $T^{\circ}\text{C.}$, may be taken to be the same as that of the liquid in the vase. The pressure, f , exerted by the water vapour corresponding to this temperature can be found from the tables, and if H is the reduced height of the barometer, the pressure to which the hydrogen is subjected is proportional to $(H-f)$.

Since the weight of 1 c.c. of dry air at 760 mm. pressure and 0°C. is .001293 grams, and the density of hydrogen is .06926, the weight of the gas liberated is given by

$$W = \frac{V \times .001293 \times .06926 \times (H-f)}{760(1 + .003665 T)}. \quad (1)$$

The average intensity of the current in amperes is therefore found from the relation

$$C = \frac{V \times 8.6525 (H-f)}{t \times 760(1 + .003665 T)}. \quad (2)$$

For very fine determinations, a correction will have to be applied to f owing to the water being acidulated; but for ordinary purposes, the tension may be taken to be the same as that exerted by the vapour of pure water.

If it is desired to produce the decomposition rapidly, the electrolyte should be so prepared as to be of maximum conductivity, its density being then about 1.25 at 15°C. It is not necessary that the liquid should stand at the same level both inside and outside the tube E when the circuit is broken. By adopting this procedure, however, the necessity of finding the density of the electrolyte after each measurement is removed.

An improved form of hydrogen voltameter is shewn in Fig. 24. The graduated tube *E* narrows down at its upper extremity until the opening is of capillary dimensions. It then expands into a little bulb, and afterwards bends over so as to rest on a support. It is filled by aspiration through the rubber tube *T*, and the capillary opening possesses the property of permitting this, and yet at the same time preventing the escape of the gas, provided that a little of the liquid is left in the bulb. A glass collar *M* attached to the vase *G*, when filled with water, keeps the gas in the tube at a uniform temperature, and enables it to be determined exactly.

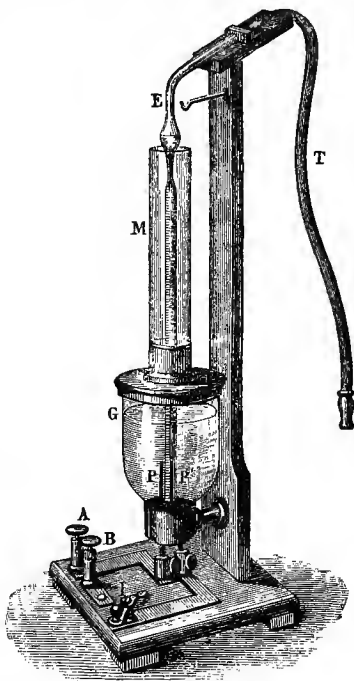


Fig. 24.

IX. COPPER VOLTAMETER.

In finding the intensity of an electric current by means of the hydrogen voltameter, errors of temperature, pressure, and volume are almost certain to be met with, and for this reason, the results obtained are often not as accurate as is desirable. When, however, current strength is measured by the amount of a metal deposited, the determinations are more accurate, since the method is one of direct weighing. It is, besides, more convenient and more cleanly in operation, and is therefore generally adopted. The metals commonly deposited are copper and silver, and the solutions employed are copper sulphate and nitrate or chlorate of silver.

When a copper voltameter is used, care must be taken to have it of such form that the density of the current, *i.e.* the quotient of the intensity by the surface of the electrodes, is very small. Otherwise, the deposit is made in little globules, which either do not adhere at all, or if they do they fall off on the slightest disturbance.

One of the best forms of the instrument is exhibited in Fig. 25, where the copper plates *A* and *B*, which are partly immersed in a solution of copper sulphate, are the positive and negative electrodes, respectively. The solution should be in the propor-

tion of five parts of water to one of copper sulphate, and should contain a slight trace of sulphuric acid.

In setting up the voltameter, both electrodes must be thoroughly cleaned by first rubbing them with fine emery paper, or dipping them in a solution of nitric acid, and afterwards rinsing them for some time in running water.

The curved plate *A* is then placed in position as indicated in the figure. *B* is well dried by

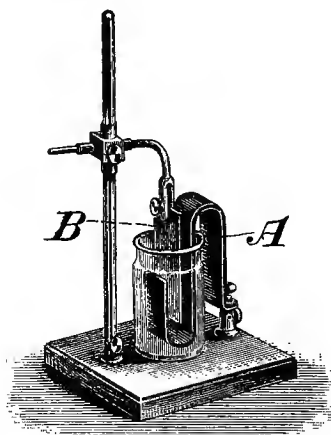


Fig. 25.

rolling it on blotting paper and then placing it in a hot-air bath. In case the time in which the experiment may be performed is limited, the plate may be rapidly dried by dipping it in strong alcohol before rolling it in the blotting paper. Any moisture which may adhere to it after this will evaporate very rapidly.

After weighing it accurately to the tenth of a milligram, this plate is attached to its support and lowered into the solution until each point on its surface is at about the same distance from some portion of the positive electrode.

The circuit should also, in this case, contain a mercurial contact breaker, and the connections be so made that the current will enter the voltameter by *A* and leave by *B*. In making a test, the current should be allowed to pass for at least fifteen minutes, and especial care should be taken to note this time accurately.

On removing the plate *B* from the voltameter after the deposit has been made, the various operations already given in regard to thoroughly cleaning and drying it are to be repeated. Care must also be taken while doing this to handle the plate very carefully, else portions of the copper may be detached.

The amount of metal deposited is found by again weighing the plate. If *W* is this amount in grams, the average intensity of the current passing during the test is given in amperes by $C = \frac{W}{t \times .000328}$, where *t* is the number of seconds the circuit was complete, and .000328 grams is the electro-chemical equivalent of copper.

Theoretically, it should make no difference in which direction the current is passed through the voltameter, as the plate *B* in the one case should lose the same amount that it gains in the other. It is found, however, that secondary chemical actions take place in the neighbourhood of the positive plate, which cause a loss in it without a corresponding deposit on the negative electrode. For this reason the amount of metal deposited rather than that dissolved is taken as a measure of the current passing.

A modification of the voltameter just described is obtained by using spiral coils instead of plates for electrodes. The two coils being of different sizes are placed one within the other in the solution, the negative generally being placed in the center. This form of the instrument possesses the advantage of presenting a very large surface to the action of the liquid.

In finding the strength of a current by the amount of silver deposited, Poggendorff's voltameter is generally used. The nega-

tive electrode is a platinum bowl resting on a metallic plate to which the leading wire is attached.

The solution is placed in this vessel, and the positive electrode, which is a disc of silver, is suspended in it so that its edge is equidistant from the sides and the bottom of the bowl.

It is customary to surround the silver disc by a small bag made of fine gauze or filter paper, so that the particles of metal which are separated by other than electrolytic action may be prevented from being deposited on the platinum.

The solution used in this voltameter contains from fifteen to twenty per cent of silver nitrate.

X. CALIBRATION OF GALVANOMETERS.

In Experiment VII. it has been shewn that when an electric current is measured by passing it through a galvanometer whose dimensions can be readily determined, its intensity is given by a relation either of the form $C = K \tan \theta$, or of $C = K \sin \theta$, according to the disposition adopted.

Many galvanometers, however, are constructed in such a manner that although their deflections follow the tangent, or the sine law, yet it is difficult and inconvenient to ascertain the number of windings in the coil, and quite impossible to obtain even an approximation to its mean radius.

The constant K , which depends on these quantities and on the intensity of the terrestrial field, cannot then be calculated, and must be determined experimentally.

METHOD I. — A standard tangent galvanometer whose constant K_1 is known is joined in series in a circuit with the one examined.

If for a given current the deflections of the two instruments are respectively θ_1 and θ , then we have

$$K = \frac{K_1 \tan \theta_1}{\tan \theta} \quad (1)$$

as a relation for determining the constant. The value of K should be found by taking the mean of a number of readings obtained by varying the resistance in the circuit, and by reversing the direction of the current through the galvanometers. When this method is followed, care must be taken to place the instruments at a considerable distance apart, else the needle of the one may be affected by the magnetic field of the other.

METHOD II.—In this method the standard galvanometer of the last is replaced by a copper or a silver voltameter of one of the forms described in Experiment IX. The current passed through is kept as steady as possible by means of a variable resistance, and any variations which do occur are corrected by taking the mean of the deflections observed at stated intervals during the test.

If θ is the mean deflection, the constant is given by $K = \frac{M}{zt \tan \theta}$, where M is the amount of the metal deposited in grams, z its electro-chemical equivalent, and t the number of seconds the current was passing. For very exact determinations the test should be continued for at least two hours, and the circuit so arranged that the deflection on the galvanometer is about 45° , this being the deflection which gives the most accurate results.

METHOD III.—This, which is a very simple method, and probably the most rapid, is a direct application of Ohm's law. A battery or cell of constant electromotive force is joined up in circuit with the galvanometer, and the current is varied by the insertion of one or more of a set of graduated resistances. If the electromotive force of the cell is denoted by E , and the deflection of the galvanometer corresponding to a given current intensity is θ , the constant is given by $K = \frac{E}{R \tan \theta}$, where R is the total resistance in the circuit. Usually the inserted resistance is very high, and in that case the resistances of the battery and the galvanometer may be neglected; otherwise these must be determined by some one of the methods given later.

It frequently happens that the current cannot be readily reduced to the proper intensity by the insertion of resistances directly in the circuit. The galvanometer should then be provided with a variable shunt, since by suitably adjusting it any desired modification of the intensity can be easily obtained. The current in the main circuit being known, that passing

through the galvanometer can then be calculated by the law of divided circuits.

In many experiments the investigation does not consist in ascertaining the intensity, but rather in detecting the presence of weak currents, and in determining their direction.

Galvanometers which are used for this purpose are of special construction, and are so designed as to be extremely sensitive.

One of the most useful of this class is that known as Thomson's (Fig. 26).

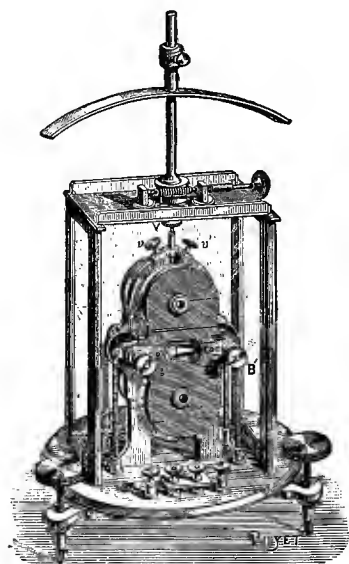


Fig. 26.

The magnets which form an astatic couple are very small, and are connected by a fine strip of aluminium, which is itself suspended from the support V by a strand of unspun silk. This strip carries a thin plate of mica for damping the oscillations of the magnets, and a small concave mirror attached to it serves to indicate the sense of the deflections.

The coils, which are so arranged that a magnet is at the center of each, are divided into two halves, and these are supported on two brass plates, one of which, LL' , is shewn in the figure. The binding poles BB' serve to connect the two parts of each coil.

A large steel bar feebly magnetized is supported above the

instrument, and as it can rotate about the vertical, or slide up and down, the earth's magnetic field and the sensibility of the instrument can be modified as desired.

On account of the great sensitiveness of this instrument it is generally used in experiments of precision. It however requires very delicate handling, and must be far removed from the presence of moving masses of iron.

Another form of galvanometer which is very sensitive, but which is more stable, is that of Depretz and D'Arsonval. It is shewn in Fig. 27.

It is the coil which moves in this instrument and not the magnet. The former is made up of a number of turns of very fine wire, carries a small

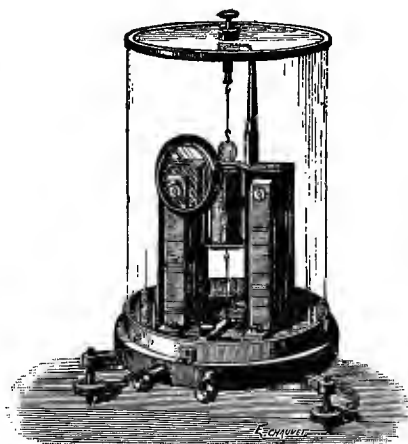


Fig. 27.

concave mirror, and is suspended by two silver wires between the poles of a horseshoe magnet, as shewn in the figure. These wires also serve to lead the current to and from the coil. A tube of soft iron, which is held within the coil, and moves with it, serves to increase the intensity of the magnetic field.

Owing to induction the damping in this instrument is so rapid that when a current is passed through the coil it almost immediately assumes its position of rest.

When observations are made with these galvanometers two different arrangements can be made. For very exact work, that shewn in Fig. 13 is adopted. There a scale is placed in front of the mirror, and the images of its graduations are observed by means of a telescope whose axis lies in the perpendicular from the center of the mirror on the scale.

Ordinarily, when rapid determinations are necessary the scale is ruled on some translucent substance such as celluloid, and is supported on a screen. In this screen there is a small opening immediately below the scale, with a dark thread stretched vertically across it. When this opening is illuminated with parallel rays of light from some source an image is formed by the mirror on the scale, and any displacement of this image will at once indicate the deflections of the instrument. In order to have a distinct image when the latter method is adopted, it is best to place the scale at a distance from the mirror equal to the length of its radius of curvature.

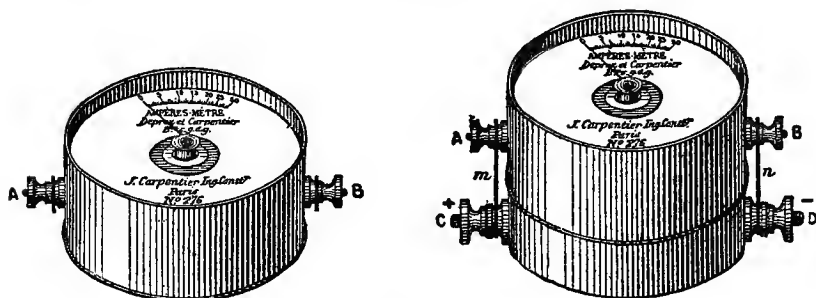


Fig. 28.

Galvanometers of this class may also be calibrated to indicate absolute intensity. Different currents are passed through the instrument, and their strengths are determined by one of the methods given above. Curves are then plotted by taking the deflections as indicated by the scale graduations for abscissæ, and the corresponding currents for ordinates. By a reference to these curves, the intensity of a current can readily be determined when the deflection it produces is known.

In Fig. 28 there is shewn one of a class of direct reading galvanometers, usually called ammeters, since they are graduated to give the value of the current strength in amperes. The principle of this instrument can be easily understood by a reference to Fig. 29.

A powerful magnetic field is formed by two semicircular permanent magnets, whose like poles are AA' and BB' . A double coil of wire CC' is fixed obliquely between the two pairs of poles, and at its center there is pivoted a small bar of soft iron, which carries the pointer indicated in Fig. 28. This bar always takes up a position with its axis coinciding with the direction of the lines of force, and when a current is passing through the coil, the magnetic field which it produces modifies that of the magnets, and the bar is therefore deflected.

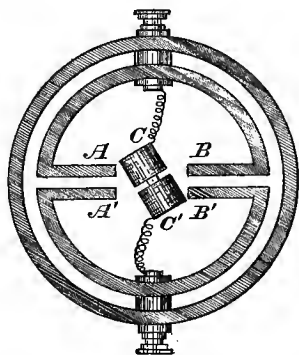


Fig. 29.

In these instruments the intensities corresponding to different deflections are recorded on the dial; and as the accuracy of the graduations depends on the magnets retaining their initial strength, they should be tested from time to time, and, if necessary, recalibrated. The range of the instrument is extended by attaching to it, by the brass strips m and n , a shunt CD (Fig. 28), whose resistance bears a simple ratio to that of the instrument proper.

XI. GALVANIC BATTERIES.

The following article is devoted to short descriptions of a few of the cells commonly used in electrical laboratories. No attempt is made to trace the development of this part of the subject, and complete details are omitted, as these can be found in any one of the many treatises dealing at length with this department of electrical investigation.

A few exercises are appended, however, for the purpose of illustrating fundamental notions in regard to the action of cells and the flow of currents. Table XX. contains the electromotive

forces of a number of elements, and Table XIX., their resistances. As the latter depend very much on the dimensions of the cell and on the strengths of the solutions used, the results are, of course, only approximate.

When possible, the laboratory should be provided with a dynamo, as currents generated in this way, especially if of strong intensity, are produced at a much smaller cost than by the use of batteries.

I. *Daniell's Cell. Meidinger type.* — In this cell there is an outer glass vessel made of two cylinders connected by a shoulder, a glass tumbler resting on the bottom of this vessel, and an inverted flask whose volume is about two liters.

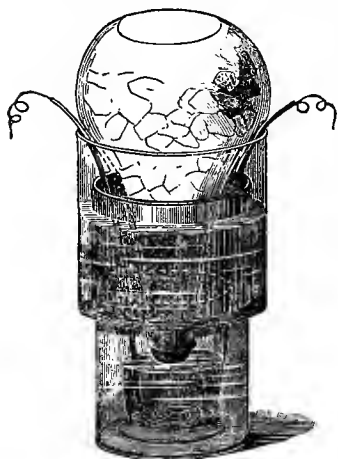


Fig. 30.

In setting up the cell, the positive pole, consisting of a copper coil or a thin plate of that metal rolled into a cylinder, is placed in the tumbler and then covered with a saturated solution of copper sulphate. The flask, after being filled as far as possible with crystals of this substance, and the balance of the space with the saturated solution, is inverted, and placed with

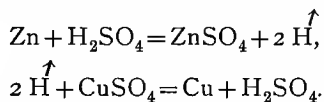
its mouth under the surface of the liquid in the tumbler. The negative pole, a cylinder of zinc, is next placed in position as shewn in the figure, and after both tumbler and flask are lowered into the outer vessel a ten per cent solution of zinc sulphate is gently poured into the latter by means of a rubber tube until it covers the zinc plate. Owing to the density of this liquid being less than that of the copper sulphate, it will, when not agitated, float on the top of the latter in the tumbler without mixing.

Wires to which clamps may be attached lead from the two

electrodes, that from the positive being insulated by gutta-percha in order to prevent its being eaten away by chemical action at the surface of separation of the liquids.

In this cell, the sulphuric acid acting on the negative electrode forms zinc sulphate and liberates hydrogen. The latter unites with the copper sulphate to form sulphuric acid, which is again ready to react upon more zinc, while the copper, which is set free, is deposited on the positive plate.

The reactions are expressed in the following equations :



In mounting this cell, and, in fact, all cells, care should be taken to have all the chemicals pure, and to use only distilled water in making the solutions. Ordinary commercial zinc contains many impurities, such as iron and lead, so that when a plate of such metal is used in a cell these particles form with the adjacent zinc a series of small voltaic elements, and local actions are set up which waste away the zinc without doing any useful work. This waste may be avoided, and the plate made to act practically as if it were pure by covering it with an amalgam. This is done by plunging it for a few seconds in dilute sulphuric acid, and then rubbing it over with mercury. After doing this it should be thoroughly washed before being used. Instead of having the positive plate of this cell all of copper, it is sometimes replaced by one of lead which has been copper-plated. This modification possesses the advantage of readily permitting the removal of the copper deposit. With ordinary usage a cell set up as described will last without renewal for about a year, and as it has a very constant electromotive force, and produces a very steady current, it is admirably adapted for laboratory work.

II. *Grove Cell*.—This element in its common form consists of a rectangular outer vessel *AE*, made of porcelain, or ebonite,

a U-shaped plate *B* of amalgamated zinc, a porous cup *C*, and a thin strip of platinum *D*.

Sulphuric acid diluted in the ratio of one to ten is placed in the outer vessel, and concentrated nitric acid in the porous cup.

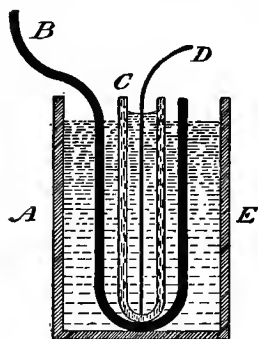
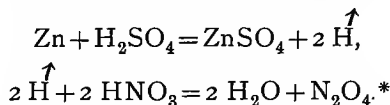


Fig. 31.

The platinum and the zinc form the positive and the negative poles of the cell respectively. The following equations express the reactions for the element :



When the cell has been allowed to act for some time the nitric acid becomes used up by the chemical action, and the sulphuric acid mixed with zinc sulphate.

There is a consequent increase in the internal resistance, but since this cell has a high electromotive force, it is capable of producing a strong current for a considerable time.

The U form of the zinc is adopted in order to thereby reduce the distance between the electrodes, but it is not very economical as the zinc is almost invariably eaten away at its lowest point first, and a large quantity of the metal is thereby rendered useless.

The chief objections to an extensive use of Grove cells arise from the high price of platinum, the injurious effects produced by the nitric peroxide gas given off, and from the nitric acid diffusing through the porous cup and acting directly on the zinc. The element is, however, an extremely convenient one, and is generally used when a strong current is required.

III. *Grenet Cell*. — This cell as ordinarily constructed consists of a large glass bottle, whose volume is about two liters, and whose shape is similar to that shewn in Fig. 32. A cover

* The oxidizing action may go much beyond this phase, and can be represented by the equation $2 \text{H}^{\uparrow} + \text{HNO}_3 = \text{H}_2\text{O} + \text{HNO}_2$.

of ebonite rests on the brass collar surrounding the upper portion of the neck, and carries two parallel slabs *C* and *C* of carbon which form the positive plates of the cell. The negative plate *Z* is made of amalgamated zinc, and is attached to a brass rod *a* which can slide freely through a socket in the cover, and so permits this electrode to be raised out of the liquid when the cell is not in use.

A piece of ebonite attached to the inside of each of the carbon plates serves to guide the zinc plate when it is being lowered, and at the same time prevents contact.

The solution for the cell is composed of bichromate of potash, sulphuric acid, and water. Various proportions have been suggested, but the following composition is considered one of the best :

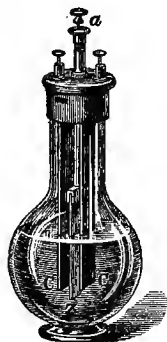
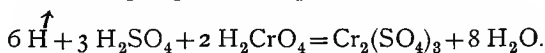
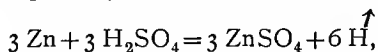
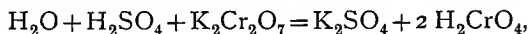


Fig. 32.

Water,	80 parts by weight.
Pulverized bichromate of potash, 12	„ „
Sulphuric acid, 36	„ „

On mixing these together chromic acid and potassium sulphate are formed, and when the zinc is lowered and the current established the chemical action consists in the chromic acid combining with the sulphuric to form chromium sulphate, and the oxygen, which is then set free, uniting with the hydrogen to form water.

The reactions are as follows :



When fresh the solution is of an orange colour, but as the potash bichromate becomes used up it changes to a green

or blue owing to the formation of chromic sulphate which combines with the potassium sulphate to form chrome alum.* As a consequence the current becomes rapidly weaker and more bichromate should then be added. If, however, the action becomes weak without this change of colour, either the supply of sulphuric acid has become exhausted, and should be renewed, or the carbons have become coated with hydrogen bubbles. These may be removed either by the chemical action of the solution if the circuit be broken for a short time, or by violently shaking up the liquid.

This element when it is first set up has a high electromotive force. It is well suited for the excitation of induction coils, but polarizes very rapidly. In some forms of the bichromate element the zinc plate is not movable, and local action is prevented by separating it from the chromic acid by means of a porous cup.

IV. *Leclanché Cell*. — The Leclanché cell in its original form is represented in Fig. 33, where *A* is a square glass vessel, *B*, the negative electrode, is a rod of amalgamated zinc, *C*, a porous

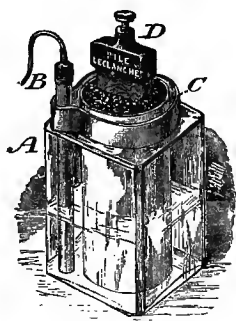


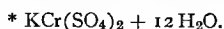
Fig. 33.



Fig. 34.

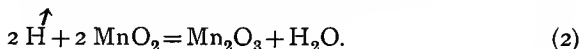
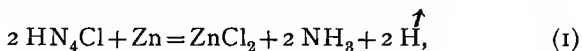
cup, contains equal quantities of peroxide of manganese and small pieces of carbon, and *D*, the positive plate, is a strip of carbon inserted in the mixture.

The exciting liquid is a saturated solution of sal ammoniac,



which about half fills the vessel *A*. When the current is passing it acts on the zinc, and forms hydrogen, ammonia, and zinc chloride. The peroxide of manganese is slowly reduced to a sesquioxide, and the oxygen given off unites with the hydrogen.

The following are the reactions :



The current from this cell falls off very rapidly on closing the circuit, but as the element has the property of rapidly building itself up, it is exceedingly well adapted for open circuit work, such as the ringing of electric bells.

The strip of carbon is generally surmounted by a lead cap, to which the leading wire may be attached, and in order to prevent the liquid rising by capillary action, and increasing the resistance of the cell by its action on the lead, the upper part of the strip is soaked in melted paraffine. This on cooling solidifies in the pores, but on its being carefully removed from the outside of the carbon, a good conducting surface is obtained, and, at the same time, capillary action is prevented.

This cell possesses many important advantages over other elements. It has a high electromotive force and a comparatively low resistance; the materials used in its construction can be obtained at a very low price; it emits no noxious gases and contains no poisonous liquids. The zinc is acted on by the sal ammoniac only when the current is passing, and, as the liquid freezes at a very low temperature, the element is much used in very cold climates.

The cell as constructed now is shewn in Fig. 34. There the porous cup is dispensed with, and the carbon electrode is held between two plates made of carbon, manganese dioxide, and shellac, by subjecting the mixture to very high pressures. The zinc is also placed close to these plates, and the whole is held together by rubber bands.

This modification has a somewhat lower internal resistance, and a slightly higher electromotive force than the cell as originally devised.

V. *Latimer-Clark's Cell*. — Many efforts have been put forth to devise some form of cell which would possess a constant electromotive force, and could therefore be adopted as a standard for comparative purposes. That proposed by Latimer-Clark more nearly satisfies the requirements than any other as yet constructed. It is prepared by dissolving sulphate of zinc in boiling distilled water. After cooling, this is decanted and the saturated solution thus obtained is used to make a thick paste with mercurous sulphate. This is heated to 100° C. in order to expel all the air, and is then poured upon the surface of warm mercury contained in some vessel of approved form. A zinc plate is then suspended in this paste, and the whole sealed in by pouring over it melted paraffine. The mercury and the zinc are the positive and the negative electrodes respectively, and each has a conducting wire leading from it, that from the mercury being of platinum, and passing up through a zinc glass tube embedded in the mixture. The reaction for the Clark cell is given by the relation $2\text{H} + \text{Hg}_2\text{SO}_4 = 2\text{Hg} + \text{H}_2\text{SO}_4$.

The electromotive forces of cells prepared in this way are found to be exceedingly uniform, their variations not exceeding .0005 volts. Owing to the possession of this property they are admirably suited for comparative work, and are greatly superior to standard Daniell cells, whose electromotive forces are found to depend largely on the concentration of the solutions used.

Careful determinations have been recently made by Messrs. Glazebrook and Skinner on this form of element, their experiments involving the examination of hundreds of cells. The results of their work shew that its electromotive force at 15° C. is 1.4342 volts. In constructing cells of this class now the paste is baked, so that on cooling it becomes very hard. This has the effect of increasing the internal resistance to about 15,700 ohms.

The cell can then be used in a closed circuit without any appreciable diminution in the electromotive force.

EXERCISE I. — *Static charge.*

Insert a plate of copper and one of amalgamated zinc in a dilute solution of sulphuric acid, and attach a wire to each of them; connect to the lower disc of a condensing electroscope the wire leading from the zinc, and to the upper disc that from the copper plate.

Then remove the wires, and lift the upper disc by means of the insulated handle. If the electroscope be very delicate, the gold leaves will diverge, shewing that they are electrified. The same phenomenon can be exhibited by interchanging the wires and repeating the operations described.

The experiment, which may be conducted much more successfully with a battery of three or four Grove cells, or a dynamo, indicates that when the circuit of a current generator is open, the two terminals are statically charged, and are at a difference of potential.

If the laboratory is provided with a quadrant electrometer, much better results can be obtained by using this instrument instead of the electroscope.

EXERCISE II. — *Polarization.*

Place, as in Exercise I., a plate of amalgamated zinc and one of copper in a vessel containing a dilute solution of sulphuric acid, and as soon as the circuit is made observe the current strength as indicated by a sensitive galvanometer. It will begin to diminish the moment the circuit is closed. If, after the cell has been allowed to act for about five minutes the circuit be opened for another five, it will be found on again closing it that it has regained almost entirely its initial intensity.

In seeking the cause of this, one has only to watch closely the action in the element. As soon as the current begins to pass, hydrogen is deposited in little bubbles on the copper elec-

trode, and, after some time, these form a gaseous layer between the sulphuric acid and the plate. If the solution be shaken up, or the plates agitated separately or together in the liquid, or the surface of the copper plate be rubbed over with a brush, it may be shewn that the diminution of the current strength depends only on the presence of this gaseous layer, and that the disappearance of the hydrogen, however brought about, from the positive electrode, is always accompanied by an increase in the current.

As the strength of the current depends only on the electromotive force of the cell and on its resistance, the presence of the gas must affect one or both of these. That it does affect the resistance is evident, since the active surface of the copper is at once diminished when it is present. It will be seen later that a diminution in the electromotive force also takes place.

In order to prevent this polarization, as it is called, cells generally contain some liquid or substance which will act chemically on the hydrogen. Examples of this are found in the use of nitric acid in the Grove, bichromate of potash in the Grenet, and peroxide of manganese in the Leclanché cell.

EXERCISE III. — *Resistance.*

In this exercise a long rectangular wooden box, a plate of copper and one of zinc, and a set of wires of homogeneous structure and of constant cross-section, are required. A solution of sulphuric acid is put in the box, and the two plates are also inserted in it at a known distance apart. Two of the wires are then selected, and these, together with a sensitive galvanometer, whose coil is also made of this wire, are used to complete the circuit. The initial intensity of the current produced is noted and then modifications of this arrangement are made by altering the distance between the plates, or the amount of their surface immersed, and also by using different lengths of wires and wires of different diameters.

If the initial intensity of the current corresponding to each disposition be carefully noted, it will be found that the results obtained go to establish the law that the current produced by a cell varies inversely as the resistance of the circuit including that of the cell, and that the resistance of a uniform conductor of any substance varies directly as its length and inversely as its cross-section.

EXERCISE IV. — *Electromotive Force.*

Many experiments establish the fact that when two electrified bodies which are at a difference of potential are brought close together, or are connected by a wire or other conductor, a discharge or current passes between them, and they are in consequence reduced to the same electrical state.

In Exercise I., it has been shewn that a difference of potential always exists between the terminals of a battery or cell in open circuit. In this case, however, the current produced when the terminals are joined does not consist of a single discharge, and is not followed by the establishment of electrical equilibrium between the two electrodes. The current is continuous and lasts as long as chemical action is kept up in the cell; while the difference of potential which this action produces between the terminals of the battery when the current is established is, owing to the existence of this current, intermediary between the potential difference in open circuit and the zero difference which obtains when the element ceases to act.

Owing to a battery or cell possessing this property of maintaining its electrodes at a difference of potential when they are connected by a conductor, it is said to be the seat of an electromotive force.

The electromotive force of an element which may thus be considered to be the cause of the current is measured by the greatest difference of potential which exists between its terminals when the circuit is open or when the circuit contains a resistance so great that practically no current passes.

It is independent of the dimensions of the cell, and depends only on the materials which enter into its composition. In order to make clear that this is so, it will suffice to perform a few experiments with cells of different types and of different dimensions.

If two zinc-copper elements, of the type described in Exercise I., be constructed exactly similar in every way, the currents produced by them will be practically of the same intensity. If, then, they be joined *in opposition*, *i.e.* so that they tend to send currents in opposite directions in the same circuit, it will be found that no current is produced. If, now, instead of having the two cells of exactly the same dimensions, the plates of one of them are made much larger than those of the other, the larger cell will (the external circuit being the same) produce much the stronger current. When the two cells are opposed to each other, however, there will again be no current produced, owing to the fact that, although the cells are of different dimensions, their electromotive forces are the same.

Further, if, instead of performing this experiment with two zinc-copper elements, one of these be used and an iron-copper element of exactly the same dimensions be taken for the other, it will be found that when these are joined in opposition there is a current produced which takes the same direction as it would if the zinc-copper element only were acting.

From this it follows that the electromotive force of the latter is greater than that of the iron-copper element, and further that the materials of which a cell is composed determines its electromotive force. In order to make this still clearer, the last experiment can be modified by using a small zinc-copper element and a very large iron-copper one. On joining these in circuit in opposition, a current is again obtained whose direction is the same as that of the current obtained in the preceding experiment. Many similar exercises can be readily devised by using different exciting liquids and plates of various other materials. These, however, will be sufficient to substantiate the statement

given above, that *it is the materials used in the construction of an element, and not its dimensions, which determine its electromotive force.*

The method adopted in these experiments can be applied with advantage to shew that when polarization takes place in an element, there is a consequent diminution in its electromotive force as well as an increase in its resistance. To do this, it is only necessary to oppose a freshly prepared cell to one exactly similar in dimensions and composition which has been allowed to generate a current for some time. The resulting current at once indicates that the former has the greater electromotive force.

XII. DETERMINATION OF RESISTANCE.

Numerous experiments shew that when two points, which are kept at a constant difference of potential, are connected by a conductor, the intensity of the current which passes depends on the dimensions of the conductor, and on the substance of which it is composed.

If the conductor is prismatic in form, and of uniform cross-section, the strength of the current varies inversely as the length and directly as the cross-section. Further, if different currents are made to pass through the same conductor by varying the potential difference of its terminals, the ratio of the current strength to the corresponding potential difference is within wide limits a constant quantity. This constant is termed the *resistance* of the conductor.

These statements which represent the laws governing the flow of electric currents are summarized in the relation called Ohm's Law, $C = \frac{E}{R}$, where C is the current intensity, E the potential difference of the terminals of the conductor, and R its resistance, which when the conductor is homogeneous is equal to $s \frac{l}{A}$, l being the length of the conductor, A its cross-section, and s a constant called the *specific resistance* of the substance composing it.

A conductor therefore possesses an absolute unit of resistance when a unit difference of potential between its terminals causes a current of unit strength to pass through it. This unit being inconveniently small for practical purposes, that chosen is the *ohm*, which is equal to 10^9 absolute units in the C. G. S. system, and is the resistance of a conductor when a difference of potential of one volt between its ends causes a current of one ampere to flow through it.

The ohm is realized practically in the resistance offered to a steady electric current by a uniform column of pure mercury 106.3 centimeters long and one square millimeter in cross-section.

Standard ohms are usually constructed of mercury, or of some metal whose resistance varies but little with slight changes in temperature.

One form of a mercurial standard is shewn in Fig. 35. The mercury is contained in a fine capillary tube, bent into a number of turns, and connected to large open tubes, also partly filled with mercury. This capillary tube is calibrated with extreme care, and in determining its length allowance is made for the resistance of the mercury in the large tubes. When in use, it is suspended as shewn in the figure, in a glass vessel filled with pieces of melted ice.

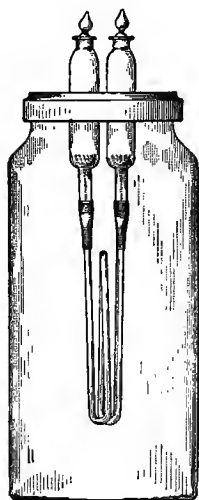


Fig. 35.

These standards are extremely fragile, and it is customary in laboratory practice to replace them by those made of metal wire. The wire, Fig. 36, is coiled in a double spiral, and its ends being attached to two bent copper rods *A*, *B*, it is then placed between two brass cylinders *E* and *C*, and the intervening space is filled with paraffine. When the standard is being used, the free ends of the

rods *A*, *B* are placed in mercury cups, which form part of

the circuit, the cylinder *C* is partly immersed in a water bath, and the temperature of the wire is ascertained by means of a thermometer inserted in the cylindrical opening *D*. The tem-

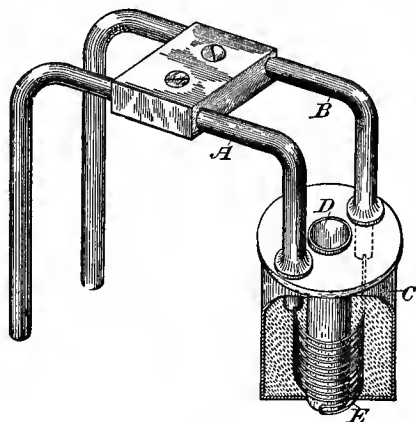


Fig. 36.

perature at which the standard is accurate is usually inscribed on it, and having been thoroughly tested, the coil is accompanied by a statement shewing its variations, with changes of

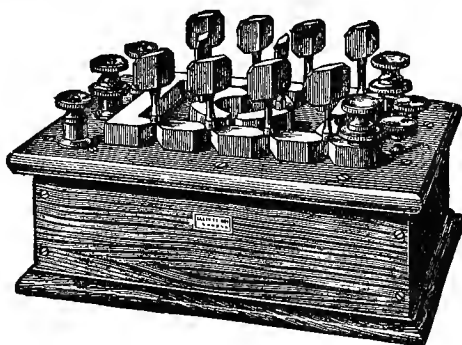


Fig. 37.

temperature. In order that these may be small the wire is usually made from an alloy of platinum and silver.

The resistance of a conductor is determined by comparison

with others whose resistances are known. These which are accurate copies of standards are generally made in coils, and are arranged conveniently in boxes provided with some device for rapidly throwing one or more of them in or out of circuit. Such an arrangement is exhibited in Fig. 37, and by reference to Fig. 38, the manner in which the coils are inserted may be readily seen. The wire corresponding to any particular resistance is bent double at its middle point, and is wound as a double spiral (to prevent induction) about either one or two bobbins, such as *A* and *B*. These are attached to a plate of ebonite, which forms the cover of the box, by means of two large brass or copper screws, which pass completely through the ebonite, and have

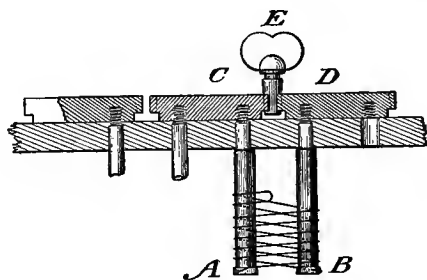


Fig. 38

their ends firmly embedded in two brass plates, *C* and *D*. These plates, which form part of the circuit, are connected electrically by soldering an end of the spiral to each of the screws. Plugs of brass, such as *E*, are made to fit snugly into grooves provided for them between the brass plates. When these are inserted the current passes directly from one plate to the other without going through the coil, since the plugs offer practically no resistance to its passage. In order, therefore, to throw any particular resistance into circuit it is only necessary to remove the corresponding plug. In the manufacture of these coils German silver is extensively used, but for very high resistances they are generally made from an alloy of platinum and silver.

Resistances of Solids. — METHOD I. — *By substitution.*

In order to determine the resistance in question, it is placed in the same circuit as a sensitive galvanometer, and a cell or battery of constant electromotive force. When the current has become steady, the deflection obtained is noted. The unknown resistance is then replaced by a box of coils, and by withdrawing the plugs resistances are thrown in circuit sufficient to reestablish the original intensity of the current, the sum of those so inserted being equal to that of the unknown.

The method is simple but primitive, and the results obtained are only approximate. It is difficult to obtain a cell which will remain constant during the experiment, and the resistance boxes ordinarily used are not graded with sufficient delicacy to be used in this way.

METHOD II. — *By use of a differential galvanometer.*

In a differential galvanometer there are two distinct coils of equal resistance, which are so wound that when currents of equal intensity are passed through them in opposite directions no effect is produced on the needle. In determining the resistance of a conductor, it is placed in circuit with one of these coils, and a rheostat or resistance box with the other. The current from a battery is made to divide so that part goes through one coil in one direction, and the remainder in the opposite direction through the other.

The needle is then deflected, and suitable resistances are thrown in circuit by means of the resistance box to bring the needle back to its initial position. The sum of the resistances so inserted is in this case also equal to that of the conductor tested. This method possesses the advantage of being sensitive and also of being independent of all variations in the intensity of the current, but it is difficult to construct a galvanometer which will have the properties assumed.

The accuracy of the instrument can be tested by noting whether there is any deflection of the needle when the same

current is passed through the galvanometer by one coil and back by the other, and also when any given current is allowed to divide freely between the two coils when their corresponding terminals are connected.

METHOD III. — *By use of a tangent galvanometer.*

A circuit is made so as to include a constant battery, a finely graduated tangent galvanometer, a rheostat, and a known resistance r , approximately equal to that of the unknown.

By means of the rheostat the intensity of the current is modified so that the needle comes to rest, making an angle of about 45° with the meridian. The unknown resistance x is then added to the circuit, and when the needle again comes to rest its deflection is noted. Finally, a reading is taken when both r and x are removed from the circuit. If α , β , and γ are the deflections in these cases respectively, we have,

$$\frac{E}{G+r} = K \tan \alpha, \quad (1)$$

$$\frac{E}{G+r+x} = K \tan \beta \quad (2)$$

$$\frac{E}{G} = K \tan \gamma, \quad (3)$$

where E is the electromotive force of the battery, and G the resistance of the whole circuit exclusive of r and x .

From (1), (2), and (3) the relation

$$\frac{r+x}{r} = \frac{\tan \alpha}{\tan \beta} \cdot \frac{\tan \gamma - \tan \beta}{\tan \gamma - \tan \alpha}$$

is obtained, and from it x can be calculated.

METHOD IV. — *The Wheatstone bridge.*

While the methods previously given are of importance in special cases, that generally adopted in laboratory practice is the method of the Wheatstone bridge. Its sensitiveness is limited only by that of the galvanometer, and it excels in admitting of rapid and extremely accurate determinations.

The principle of the method is illustrated in Fig. 39.

The main circuit of a battery E divides at A into two branches, ABC and ADC , which contain the resistances a , x and b , c , respectively. Between the terminals of these resistances B and D a galvanometer G is inserted, and as a current of short duration only is required, both this circuit and the main one contain contact keys to permit the current to be established or broken at will.

In making a test the unknown resistance x is inserted between B and C , and the other resistances a , b , and c are so adjusted

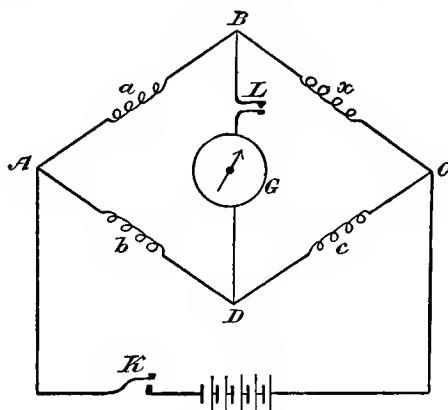


Fig. 39.

that on depressing the key L , no current passes through the galvanometer. The points B and D are then at the same potential. Denoting it by V_2 , and the potentials of the points A and C by V_1 and V_3 respectively, it follows from Ohm's law, since the intensity of the current in each of the circuits ABC and ADC is the same throughout its length, that

$$\frac{V_1 - V_2}{a} = \frac{V_2 - V_3}{x}, \quad (1)$$

and
$$\frac{V_1 - V_2}{b} = \frac{V_2 - V_3}{c}. \quad (2)$$

Dividing each member of (1) by the corresponding one of (2), we obtain the relation

$$\frac{a}{b} = \frac{x}{c} \text{ or } x = \frac{a}{b} c.$$

If, therefore, the resistance of a or c , and either the resistances of the other two, or their ratio, be known, that of x can be calculated. It is best to take b equal to a at first, and then to vary c until by trial it is approximately equal to x . By

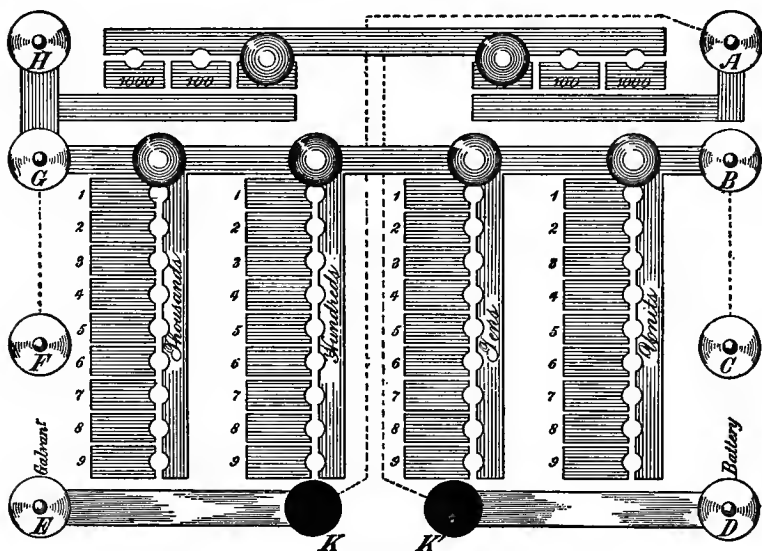


Fig. 40.

then giving a a certain value, and gradually increasing b , that of x can be determined with a degree of accuracy which is limited only by the resistances available, and as already indicated by the sensitiveness of the galvanometer.

A practical form of the bridge consists of a resistance box containing three sets of coils. One of the best arrangements is that shewn in Fig. 40, where a and b of the theoretical diagram are represented by two sets of ten, one hundred, and one thousand ohm coils. These are inserted between the brass

plates bearing these numbers, and the two brass strips shewn in the figure connected to A and H . In order to place any one of them in circuit it is only necessary to insert a plug in the corresponding aperture. The third resistance c is given a wider range, and consists of thirty-six coils placed between B and G , and graded in powers of ten so as to give any resistance from 1 to 9999 ohms.

The unknown resistance is placed between A and B , the galvanometer between F and E , and the battery between C and D . The two keys, K' and K , correspond to those in Fig. 39 indicated by K and L respectively, and the dotted lines exhibit the paths of the current through the instrument.

Besides being exceedingly compact, this form of bridge possesses the advantages of having a minimum number of plugs, and of permitting readings to be made directly. In determining a resistance care should be taken to depress the battery key K' before that of the galvanometer K , and to release them in the reverse order, else the disturbing effects of induction currents will be experienced and unnecessary delay be caused.

When a preliminary test is being made the galvanometer key should not be pressed down firmly, otherwise the contact will be good, and the galvanometer needle may be so violently deflected as to break the suspending fiber. On account of its constant electromotive force, a Daniell cell is peculiarly suited for work on the Wheatstone bridge. If, however, such a cell is not available, others may be used, and in case the current is too strong part of it may be short-circuited by the insertion of a shunt.

In comparing the conducting powers of different materials it is necessary to know their *specific* resistances. In the case of metals, or other solids, these can be readily determined for any temperature by finding the resistances of long uniform wires or rods of the substances, and then applying the relation $s = R \frac{A}{l}$

Resistances of Liquids. — In determining the specific resistance of a liquid such as mercury a capillary tube is attached by rubber tips to two small steel or wooden cups which have short tubes protruding from their sides.

The liquid in one of these cups is forced through the capillary tube, expelling the air, and is made to gradually fill up the cup to which the other end is attached.

Terminal wires are then inserted in the mercury in the cups, and the resistance, which will be practically that of the liquid in the capillary tube, is determined by one of the methods given above. It only remains to calculate the dimensions of this tube and to apply the relation already given.

For very accurate results the tube must be carefully calibrated by the method adopted in the construction of thermometers, but for ordinary purposes that explained on page 32 may be followed.

In order to insure a uniform temperature for making the test, the cups and the capillary tube should be placed in a water or oil bath.

In case the liquid is an *electrolyte*, a serious difficulty is met with in the polarization of the electrodes. On passing a current through such a liquid, its resistance is altered owing to changes in its condition near the electrodes, and to the presence of the chemical products deposited there.

The existence of an electromotive force opposed to that producing the current still further complicates the problem, and renders an accurate determination of the resistance difficult.

It has been found when the liquid examined is a solution of a metallic salt, that if the electrodes are composed of the metal forming the base of this salt the polarizing effects are small, and the resistance of the liquid may be found approximately by the ordinary methods.

One of the early devices adopted was to place the liquid in a rectangular trough, and to have movable electrodes inserted in it. The resistance of a certain length of the liquid was first

determined, and then the distance between the electrodes being shortened by a known length, the current was restored to its original intensity by means of a rheostat placed in the circuit. The amount of the wire resistance thus added was taken as a measure of the resistance of the liquid through which the electrode was moved.

Alternating currents have been applied with considerable success to finding the resistances of electrolytes. A Wheatstone bridge of special construction is used, but the theoretical arrangements are the same as have already been described.

An electro-dynamometer, however, takes the place of the galvanometer, and this instrument again is sometimes replaced with advantage by a telephone. When the latter is used, and the alternating currents are produced by means of a rapidly vibrating induction coil, a humming sound is heard, which, when the proper proportions obtain among the resistances, weakens to a minimum and nearly disappears.

In this method polarization is avoided since the current is passed through the liquid alternately in opposite directions.

XIII. TEMPERATURE COEFFICIENT OF RESISTANCE.

SLIDE WIRE BRIDGE.

The resistance of a conductor depends not only on its dimensions, and on the substance of which it is composed, but also on its temperature. Owing to the extensive use of resistance coils for comparative purposes, it becomes therefore a matter of great importance to select materials for their construction which vary but little with temperature, and to know exactly the law governing such variations.

The resistances of alloys such as platinum silver, German silver, gold silver, and platinoid, increase much less rapidly with a rise in temperature than do pure metals, while carbon forms a notable exception to the general law for elemental substances in that its resistance decreases as its temperature rises.

In determining a law for this variation in the resistance of metals a wire of suitable length of the substance in question is selected, and a series of experiments are performed to ascertain exactly its resistances at different temperatures.

By means of the results obtained the law is then exhibited graphically by plotting a curve, taking temperatures for abscissæ, and resistances for ordinates; but for the purposes of calculation it is better to follow the method of Least Squares, and to establish an empirical formula connecting the resistances of the conductor with its corresponding temperatures.

This relation is usually obtained in the form

$$R = R_0(1 + at \pm \beta t^2),$$

in which R_0 is the resistance of the wire at zero, and the coefficients a and β are calculated from the observed results.

As these coefficients depend only on the substance of the wire, this relation can therefore be applied to ascertain the resistance of any conductor of the same material at any given temperature when its value at 0° C. is known. In order to arrive at results which will be reliable, every precaution must be taken to measure the resistances of the wire accurately, and to note its temperatures with the greatest precision.

The wire, which should be well insulated, is wound on a thin hollow bobbin made of wood, and is inserted in a thin glass tube closed at one end. A delicate thermometer is also placed in this tube, and the whole is suspended in a liquid bath which should be provided with a stirrer, and a second thermometer to check the readings of the first.

The resistance of the wire is generally determined by means of a Wheatstone bridge, the connections being made by soldering two stout wires to its terminals.

When a test is about to be made a Bunsen flame is applied to the bath until it reaches a suitable temperature. It is then withdrawn, and the liquid is stirred until the two thermometers shew that its temperature has become steady. The resistance

is then determined, and the corresponding temperature of the wire is found by taking the mean of the readings of the two thermometers. More heat is then applied, and the same operations are repeated until the whole range of temperatures chosen has been covered.

If it is desired to make this somewhat extensive, an oil bath should be used, but as it emits an exceedingly offensive odour it is better for ordinary purposes to use water instead of oil. To increase the accuracy of the work the test should be conducted with falling as well as rising temperatures.

The errors which necessarily accompany the results obtained by using the ordinary form of the Wheatstone bridge cannot be

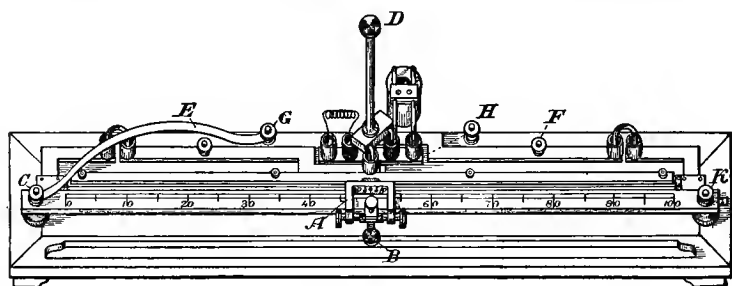


Fig. 41.

neglected in a delicate experiment such as this, and a simplified form of the instrument known as the slide wire bridge is generally used.

In connection with Fig. 39, it has been shewn that when the points *B* and *D* are at the same potential, the resistances are connected by the relation $x = \frac{a}{b}c$. In the slide wire bridge, a single wire of homogeneous structure and of constant cross-section takes the place of the resistances *b* and *c*, and in determining that of *x*, the galvanometer terminal is moved along this wire until a point *D* is reached, which is at the same potential as that of *B*.

Since the resistance of any portion of this wire is proportional to its length, that of the unknown is given by $x = \frac{l_1}{l_2}a$, where l_1 and l_2 are respectively the lengths AD and DC of the wire AC .

In its practical form, the bridge is shewn in Fig. 41. The wire, which may be of brass or an alloy of platinum, is one meter long, and is tightly stretched between the points C and K . A heavy brass bar, on which is ruled a meter scale with centimeter divisions, is so placed in front of this that the terminals of the wire are opposite the limiting divisions of the scale.

A slider A provided with a vernier, and with a key B for making contact with the wire, can move freely along this bar, and a series of brass strips, shewn in the figure, supply the remaining connections.

The unknown resistance and a standard are introduced into the circuit by means of four mercury cups, two of which are of brass and two of ebonite, and the relative positions of these can be reversed by means of the commutator D , constructed of two U-shaped brass or copper rods, whose terminals also rest in the mercury cups.

The battery terminals are attached to the bridge at E and F , and those of the galvanometer to the binding poles G and H , which are connected to the graduated brass bar and to the central metallic strip supporting the mercury cups respectively. The slider A can be clamped in any position, and the key is so arranged that by depressing it a spring is acted on which causes a rounded metallic knife-edge to press gently but uniformly against the stretched wire.

Best results are obtained when l_1 and l_2 are nearly equal, and the unknown resistance, therefore, should be always so selected that it is at 0° C. approximately equal to that of the standard.

Care must be taken not to touch the wire or the other metal-

lic parts of the bridge by the hands during a test, otherwise thermo-electric currents are at once set up, which greatly disturb the readings.

The slide wire bridge is especially useful in the construction of resistance coils which are intended to be very accurate copies of standards.

XIV. GALVANOMETER RESISTANCE.

If the laboratory is provided with a second galvanometer sufficiently sensitive, the coils of the one to be examined may be treated as an ordinary conductor, and their resistance determined by one of the methods outlined in the last experiment.

If, however, a second instrument is not at hand or is not conveniently adjusted, either the method of equal deflections or that known as Thomson's may be applied, and in these tests, the galvanometer whose resistance is required is itself used as the current indicator.

(1) *Method of equal deflections.*

The arrangement is that shewn in Fig. 42. The current which is supplied by a constant battery E divides at C , part

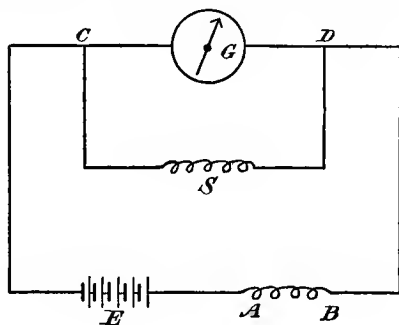


Fig. 42.

going through the galvanometer G , and part through a shunt S . The main circuit also contains a variable resistance AB .

Denoting the resistance of the galvanometer by g , that of the shunt by s , and that of the battery and leading wires by r , we have, when the resistance of AB is R_1 , the intensity of the main current given by

$$C = \frac{E}{r + R_1 + \frac{gs}{g+s}}, \quad (1)$$

and that of the part passing through the galvanometers by

$$\begin{aligned} C_1 &= \frac{E}{r + R_1 + \frac{gs}{g+s}} \times \frac{s}{g+s} \\ &= \frac{Es}{(r + R_1)(g+s) + gs} \end{aligned} \quad (2)$$

If now the shunt be removed from the circuit, and the resistance of AB altered until the galvanometer indicates that the original intensity of the current passing through it is restored, the strength of the current is given by

$$C_1 = \frac{E}{r + R_2 + g}, \quad (3)$$

where R_2 is the new value of AB .

Equating the right-hand members of (2) and (3), it follows that the resistance of the galvanometer may be found from the relation,

$$g = \frac{s(R_2 - R_1)}{R_1 + r}. \quad (4)$$

As the theory indicates, it is absolutely necessary for a successful application of this method to use only a battery which will remain constant for a considerable time. The method is somewhat tedious, owing to the necessity of previously determining the internal resistance of the battery, and best results are obtained when a battery is selected whose resistance is so small that it can be neglected.

In that case, if the resistance of the leading wires is also neglected, that of the galvanometer is given by

$$g = \frac{s(R_2 - R_1)}{R_1} \quad (5)$$

(2) *Thomson's method.*

This is an application of the principle of the Wheatstone bridge, similar to that described in Experiment XII. The galvanometer is disposed, as shewn in Fig. 43, between the two points *C* and *D*, and a contact key is placed in the circuit between *C* and *B*.

If, when the key is raised and a current is passing through the galvanometer, the points *C* and *B* are not at the same

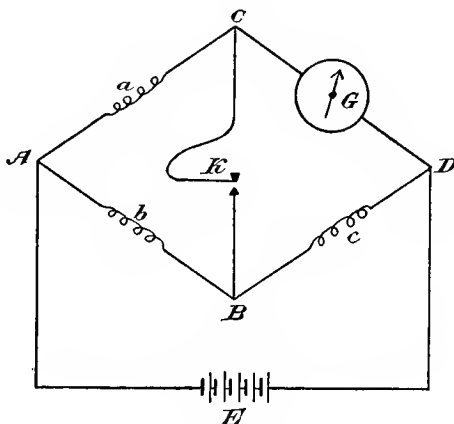


Fig. 43.

potential, then a current will pass between these points when the key is depressed, and a consequent alteration in the deflection of the galvanometer will occur.

If, however, the resistances *a*, *b*, and *c* are so adjusted that *C* and *B* are at the same potential, it is quite immaterial whether the key is raised or depressed, as no current will pass between these points, and there will therefore be no change

in the intensity of the current passing through the galvanometer.

When this condition obtains the ordinary relation for the equilibrium of the Wheatstone bridge applies, and the resistance of the galvanometer is given by

$$g = \frac{a}{b}c. \quad (6)$$

The method is exceedingly simple, and is preferable to the former, in that it is quite independent of the resistance of the battery.

In case the battery used does not remain constant during the experiment, the deflection of the galvanometer needle will gradually change. This, however, will not affect the application of the method, as the test consists entirely in noting whether the intensity of the current passing through the galvanometer is affected by raising or depressing the contact key.

In case the current through the galvanometer produces too great a deflection, a shunt should be inserted between the points *A* and *D*. By giving different values to this, the current passing through the bridge may then be suitably modified.

XV. RESISTANCE OF BATTERIES OR CELLS.

From the exercises on batteries in Experiment XI., it is evident that a cell or battery acts in the same manner as an ordinary conductor, and offers a resistance to the passage of the current which it produces.

The value of this resistance depends to some extent on the electrodes of the cell, on the amount of their surfaces submerged, and on the distance they are apart. It also varies within very wide limits with the composition of the battery solutions, and with their concentration.

It, however, cannot be considered a definite quantity, since polarization occurs in most cells as soon as the current is established, and the resistance gradually increases, its value at any instant depending both on the strength of the current then

passing and on the length of time the circuit has been closed. In determining the value of this quantity, therefore, for a current of given intensity, the result obtained can be taken only as an approximation to its correct value under other circumstances ; but such determinations, however, are sufficient for ordinary purposes, and may be made by one or other of the following methods :

(1) *Ohm's method.*

This method is a direct application of Ohm's law to a complete circuit, including the battery to be tested, a sensitive galvanometer, and a variable resistance R .

Denoting the electromotive force of the cell by E , the resistance of the galvanometer and leading wires by g , and that of the cell by r , the relation

$$C_1 = \frac{E}{r+g+R_1} \quad (1)$$

will represent the intensity of the current produced when R_1 is the resistance given to R . On changing the variable resistance to R_2 , a second relation

$$C_2 = \frac{E}{r+g+R_2} \quad (2)$$

is obtained ; and, assuming that E remains constant during the test, it follows from (1) and (2) that

$$C_1(r+g+R_1) = C_2(r+g+R_2),$$

$$\text{or} \quad r = \frac{C_2 R_2 - C_1 R_1}{C_1 - C_2} - g. \quad (3)$$

If the readings of the galvanometer follow the tangent law, this relation becomes

$$r = \frac{\tan \theta_2 R_2 - \tan \theta_1 R_1}{\tan \theta_1 - \tan \theta_2} - g,$$

in which θ_1 and θ_2 are the deflections corresponding to the currents C_1 and C_2 .

In adopting this method a number of readings should be taken by varying the resistance R , and in each case the results should be checked by reversing the current through the galvanometer.

(2) *Thomson's method.*

As shewn in Fig. 44, the battery E , whose resistance is required, is joined in series with a galvanometer G and a variable resistance AB , while a shunt S is inserted between the points F and H .

In applying the method a suitable deflection is first given to the galvanometer needle by properly adjusting the resistances AB and S . The shunt is then removed, and the rheostat

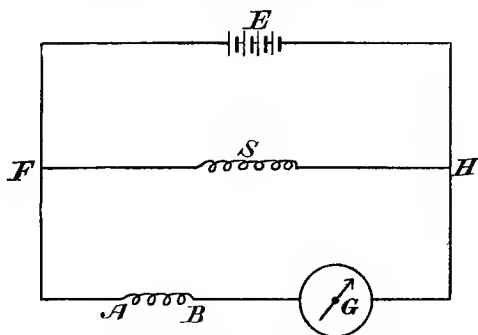


Fig. 44.

altered, until the original intensity of the current passing through the galvanometer is reestablished.

If in the first operation s and R_1 are the values given to S and to AB , and in the second R_2 that given to the rheostat, the expressions for the current passing through G in the two cases are,

$$C = \frac{E}{r + \frac{(R_1 + g)s}{R_1 + g + s}} \times \frac{s}{R_1 + g + s}, \quad (1)$$

and
$$C = \frac{E}{r + R_2 + g}. \quad (2)$$

By equating the right-hand members of these equations, the relation $r = \frac{s(R_2 - R_1)}{R_1 + g}$ may be deduced, and from it the internal resistance of the battery may be calculated.

(3) *Mance's method.*

In applying the previous methods considerable time is required to make the two adjustments, and unless the element examined polarizes very slowly, the results obtained cannot be taken as accurate.

In Mance's test, however, the time which elapses between the two observations is exceedingly short, being merely that spent in closing a circuit, and it is therefore not sufficient for the battery resistance to undergo any perceptible change. For

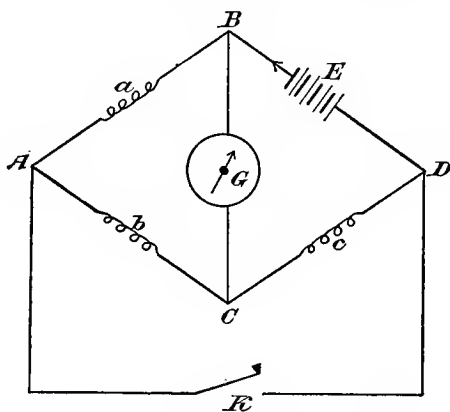


Fig. 45.

this reason this method is generally followed in determining the resistance of a cell, or battery, which polarizes very rapidly.

The arrangement is that shewn in Fig. 45. There the cell is inserted in one of the arms of the bridge, the galvanometer in one of its diagonals, and a coarse wire of very small resistance, containing a contact key in the other.

The test consists simply in so adjusting the resistances a , b , and c that no difference is observed in the strength of the current passing through the galvanometer when the key is raised or depressed.

When the resistances are so disposed that the proper conditions obtain, they are connected by the relation $r = \frac{a}{b} c$, where r is the resistance of the cell.

This may be shewn by considering the currents passing through the galvanometer under the two arrangements. In the first the current divides at B , and takes the paths BAC and BC to reach the point C . The strength of the current in this case is given by

$$C_1 = \frac{E}{r + \frac{g(a+b)}{a+b+g} + c} \cdot \frac{a+b}{a+b+g} \quad (1)$$

In the second, depressing the key amounts to the same thing as bringing the two points D and A together. Denoting this hypothetical point of union by DA , the current may be considered as dividing at B , and then passing partly by BA to DA , and partly by BC , and its subdivisions CD and CA to the same point, and thence back to the battery.

Under these circumstances the current intensity is given by

$$C_2 = \frac{E}{r + \frac{a\left(g + \frac{bc}{b+c}\right)}{a+g + \frac{bc}{b+c}}} \times \frac{a}{a+g + \frac{bc}{b+c}} \quad (2)$$

If, then, C_1 is equal to C_2 , it will be found on equating the right-hand members of (1) and (2) that the resistances satisfy the relation

$$r = \frac{a}{b} c.$$

This can also be readily seen by considering the intensities of the currents in the various branches of the bridge when the

key is depressed, as being the resultant of two steady systems, the first being that which exists when the key is raised, and the second being some system in which the points *B* and *C* are at the same potential. As is evident from Method IV., Experiment XII., this can be considered to be an arrangement in which an electromotive force is inserted in the circuit *AKD*, and in which consequently the resistances of the arms of the bridge are connected by the formula

$$r = \frac{a}{b} c.$$

In applying this method it will be found that in most cases the galvanometer deflection will be inconveniently large. This defect may be remedied by shunting the galvanometer, or if the instrument is provided with a directing magnet, by so disposing it that it will act in an opposite direction to that of the current, and so reduce the deflection.

If the laboratory is provided with a potential galvanometer, or finely graduated voltmeter, the resistance of a cell may be determined by observing the differences of potential between the terminals of the element first in open circuit, and then when the circuit is completed by a small known resistance *R*. Denoting these differences of potential by *E* and *V* respectively, it follows from Ohm's law that

$$\frac{E}{r+R} = \frac{V}{R},$$

or that

$$r = \frac{E-V}{V} \cdot R.$$

XVI. DETERMINATION OF ELECTROMOTIVE FORCES.

In Experiment XI. a few exercises are indicated as being suitable for giving clear and accurate notions regarding the electromotive force of a cell or battery. The following article is devoted to a description of some of the best methods that may be followed in determining this constant for a given element.

As already stated, the electromotive force of a cell is measured by the greatest difference of potential between its terminals in open circuit. The unit potential difference in the C. G. S. system is inconveniently small for practical purposes, and that generally adopted is the *volt*. It is equal to 10^8 units in the C. G. S. system, and is the difference of potential that must be maintained between the terminals of a conductor of one ohm resistance in order that the intensity of the current produced in it may be one ampere.

While the first three methods are for comparative determinations by reference to some standard cell whose constant is known, the following ones indicate how an absolute measure of the electromotive force of an element may be obtained in terms of resistance and current intensity.

I. *Comparative determination of electromotive forces.*

(1) *Wheatstone's method.*

A standard cell, such as that of Daniell or Clark, is joined in simple circuit with a galvanometer, and a rheostat, or resistance box. The latter is adjusted to produce a convenient deflection of θ° , and then a known resistance r is added to the circuit, and a new deflection of θ_1° is obtained.

The standard is next replaced in the circuit by the cell, whose electromotive force is required, and by means of the rheostat the current is modified until the initial deflection of θ° is reproduced. It only remains then to determine what resistance r_1 must now be added to reestablish the second deflection of θ_1° , and from these known quantities the unknown electromotive force E_1 can be calculated in terms of that of the standard E .

Denoting the resistances of the whole circuit in the two cases, corresponding to the deflection of θ° by R and R_1 , it follows from Ohm's law, that

$$\frac{E}{R} = \frac{E_1}{R_1}, \quad (1)$$

and that

$$\frac{E}{R+r} = \frac{E_1}{R_1+r_1} \quad (2)$$

By combining these we have,

$$\frac{E_1}{r_1} = \frac{E}{r},$$

and therefore the electromotive force E_1 is $\frac{r_1}{r}$ times that of the standard.

The method is simple, and is very accurate if the cells used are constant. If not, however, the method cannot be successfully applied.

(2) *Lumsden or Lecoine's method.*

The arrangement for this method is shewn in Fig. 46. The standard A and the cell to be tested, B , are mounted in series

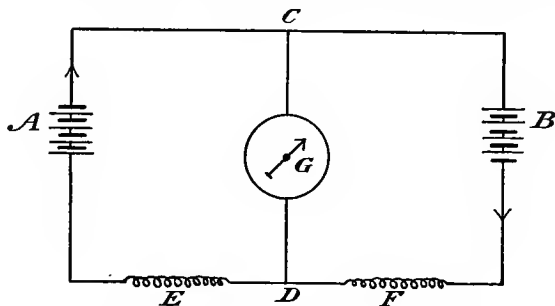


Fig. 46.

with their opposite terminals connected in a circuit, which also contains the variable resistances E and F .

A shunt, which includes a sensitive galvanometer, connects the points C and D .

In applying this method a certain value is given to one of the adjustable resistances, and then the other is varied until no current passes through the galvanometer. If E and E_1 denote the electromotive forces of the cells A and B respectively, and r and r_1 their internal resistances, these quantities are connected by the relation

$$\frac{E_1}{E} = \frac{R_1 + r_1}{R + r}, \quad (1)$$

R and R_1 being the resistances of E and F respectively (including that of their connecting wires). This will be readily seen from a consideration of the following theory: In all such cases as this where the circuit has many branches, and contains a number of electromotive forces, the *principle of superposition* is to be applied. Each electromotive force in combination with the others has precisely the same effect as it would have were it the only one in the circuit, and hence the problem of determining the current passing in any one of the branches can be greatly simplified by considering the action of each of the cells separately.

In the above arrangement the two cells tend to send a current through the galvanometer circuit in opposite directions, and since no deflection is produced in this instrument, the currents which they would send through, were they acting separately, must then be equal.

That which would pass through it, were the standard the only one acting, is given by

$$C = \frac{E}{r + R + \frac{g(r_1 + R_1)}{g + r_1 + R_1}} \times \frac{r_1 + R_1}{g + r_1 + R_1}; \quad (1)$$

while if the cell B were the only one producing a current, the amount which it would send through the galvanometer is given by

$$C_1 = \frac{E_1}{r_1 + R_1 + \frac{g(r + R)}{g + r + R}} \times \frac{r + R}{g + r + R}. \quad (2)$$

If, then, C is equal to C_1 , it follows that

$$E(r_1 + R_1) = E_1(r + R),$$

or that

$$\frac{E_1}{E} = \frac{r_1 + R_1}{r + R}. \quad (3)$$

If, now, r and r_1 are known, E_1 can be at once found in terms of E , but if not, a second observation is made by increasing R

by some value ρ , and then determining what resistance must be added to R_1 to again reduce the current in CD to zero. Denoting this by ρ_1 , we have,

$$\frac{E_1}{E} = \frac{r_1 + R_1 + \rho_1}{r + R + \rho}, \quad (4)$$

and combining (3) and (4), it follows that

$$\frac{E_1}{E} = \frac{\rho_1}{\rho}, \text{ or } E_1 = \frac{\rho_1}{\rho} E.$$

This method, just as the preceding one, can only be applied successfully to constant elements. If, however, one of the cells is constant, it affords a means of studying the process of polarization in the other. The special advantage of the method is that both cells are working under exactly the same conditions.

(3) *Poggendorff's method.*

This, which is probably the most exact of all the methods devised, involves a new idea, that of determining the electromotive force of an element by balancing it against a potential difference maintained in a circuit by means of a standard cell.

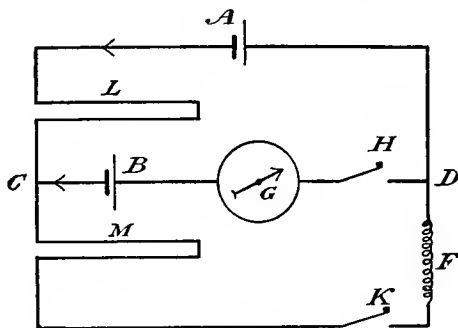


Fig. 47.

As Fig. 47 indicates, the standard A is joined in series with a rheostat F , and two carefully constructed sliding resistances L and M , B the cell to be tested together with a galvanometer

G , being inserted as a shunt between the points C and D in such a manner that its current tends to oppose that produced by the standard cell. Contact keys are provided at K and H , and the various connections of the circuit are made of stout brass or copper strips, in order that their resistance may be negligible.

When the arrangements described have been made the rheostat F is varied until on depressing the contact keys the galvanometer shews that no current is passing along CD .

This condition implies that a difference of potential exists between the points C and D exactly equal to the electromotive force of the cell B .

The accuracy of this adjustment can be tested by first giving the rheostat F a value slightly above that which it then has, and afterwards one slightly below it. This should result in the galvanometer indicating a current passing first in one direction, and then in the other along CD .

After this, a known resistance R is thrown in circuit by means of the sliding resistance M , and then a corresponding amount R_1 is added at L to reproduce the condition of zero current in CD .

The electromotive force of the element B is then given in terms of that of the standard E by the relation

$$E_1 = E \cdot \frac{R}{R + R_1}. \quad (1)$$

This follows from first applying Ohm's law to the whole circuit $ACKD$, and then to the part CKD . Denoting the potential difference between C and D by $V_1 - V_2$, the resistance of F by ρ , and that of the battery A by r , we have, for the first adjustment,

$$\frac{E}{r + \rho} = \frac{V_1 - V_2}{\rho}, \quad (1)$$

and for the second,

$$\frac{E}{r + \rho + R + R_1} = \frac{V_1 - V_2}{\rho + R}. \quad (2)$$

Combining (1) and (2), we have,

$$\frac{V_1 - V_2}{E} = \frac{R}{R + R_1},$$

and since the potential difference $V_1 - V_2$ is equal to the required electromotive force E_1 the relation given previously is evident.

Owing to the determination being made with the cell practically in open circuit, errors due to polarization cannot arise, and the method can therefore be readily applied to all classes of cells, including those which polarize even very rapidly. The standard selected should be a constant element, but even should it vary, errors due to this cause may be avoided by closing the circuit only for short intervals at each test. In case the galvanometer used is a reflecting one, it will suffice to merely depress the keys K and H for an instant. If the cell tested is not properly balanced, the momentary contact will be sufficient to indicate this.

Like the preceding, this method is well adapted for studying the effects of polarization in a cell by measuring its electromotive force after it has been allowed to produce currents of different intensities for varying intervals of time.

As the theory indicates, the electromotive force of the standard must be higher than that of the element tested; if, however, it is lower, the method can still be applied by joining a number of standard cells in series until an electromotive force sufficiently high is obtained.

(4) *Clark's method.*

This method is based on the same principle as the last, but it is so modified that the standard and the unknown element are compared under precisely similar circumstances.

The disposition is that outlined in Fig. 48. The current is produced by a constant battery C , of resistance r , whose circuit $CDEF$ contains a rheostat M , and a sliding resistance EF .

The standard A , together with a galvanometer, is inserted as a shunt between the points D and P , and B the cell to be examined and another galvanometer are inserted in a second derived circuit EKN .

The standard cell is first balanced by properly adjusting the resistance of the rheostat, and then the contact key K is

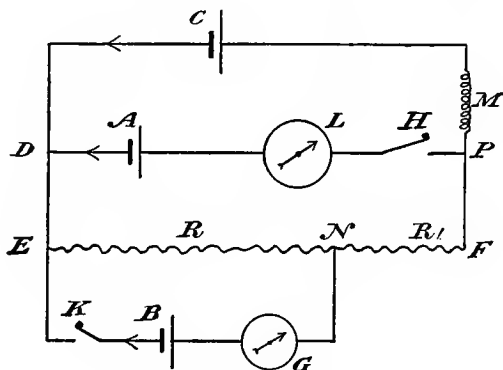


Fig. 48.

depressed and the terminal N is moved along EF , until a point is reached when no deflection is observed in the galvanometer G .

The cells A and B are then both balanced, and the current is produced by the battery C alone.

Denoting the electromotive forces of A , B , and C by E , E_1 , and E_2 respectively, the resistances of the two parts of EF by R and R_1 , and that of the rheostat by ρ , we have,

$$\frac{E_2}{r + \rho + R + R_1} = \frac{E}{R + R_1}, \quad (1)$$

and

$$\frac{E_2}{r + \rho + R + R_1} = \frac{E_1}{R}. \quad (2)$$

The unknown electromotive force is then given by the relation,

$$E_1 = \frac{R}{R + R_1} \cdot E. \quad (3)$$

Here, again, the electromotive force of C must be greater than of either of the elements A or B , and in case E_1 is greater than E , the dispositions of these two elements must be interchanged. The sliding resistance EF is generally made of platinum-iridium wire of constant cross-section, and it is so arranged that it can be minutely subdivided. That employed by Latimer-Clark in his investigations was of 40 ohms resistance, and was so wound on an insulated cylinder that it could be read to one twenty-thousandth of its length.

II. *Absolute determination of electromotive forces.*

From the relation $C = \frac{E}{R}$, it is evident that if C and R be expressed in absolute units, a value for E can at once be deduced in the same system.

So far as C is concerned, this can be readily accomplished by the use of a tangent galvanometer, but in the case of R , since it includes the internal resistance of the cell, difficulties arise in connection with direct determinations which make the results so obtained practically worthless. Hence, methods are adopted in which a determination of the internal resistance is avoided.

For constant cells Ohm's method may be followed, but it is better both with constant and variable elements to apply that outlined by Latimer-Clark, which is a method of *compensation*, the cell being balanced against a difference of potential maintained by an auxiliary generator.

(1) *Ohm's method.*

The circuit consists of a rheostat, or resistance box, a tangent galvanometer, and the element whose electromotive force is to be determined.

A convenient deflection θ_1 is given to the galvanometer by suitably adjusting the rheostat, and then a known resistance R is added to the circuit, and a second deflection θ_2 is obtained.

Denoting the resistance of the circuit corresponding to the first deflection by ρ , it follows that

$$\frac{E}{\rho} = K \tan \theta_1, \quad (1)$$

and
$$\frac{E}{\rho + R} = K \tan \theta_2, \quad (2)$$

K being the constant of the galvanometer.

Eliminating ρ from these equations, the relation

$$E = \frac{KR \tan \theta_1 \tan \theta_2}{\tan \theta_1 - \tan \theta_2}$$

is obtained, and from it E can be found in absolute units.

(2) *Clark's method.*

The disposition adopted by Latimer-Clark is similar to that devised by Poggendorff for comparative measurements. A tangent galvanometer G (Fig. 49), whose constant K is known,

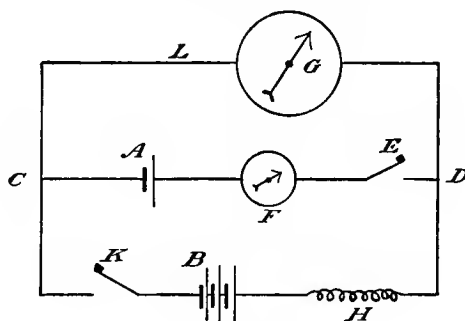


Fig. 49.

is inserted along with a rheostat H in the circuit of a constant battery B , and the cell A , whose electromotive force is to be found, together with an auxiliary galvanometer, form the derived circuit CD .

The corresponding terminals of the two cells are connected, so that they both tend to send a current through the galvanometer G in the same direction.

The rheostat is varied until the galvanometer F indicates that the electromotive force of A is balanced by the potential difference between the points C and D . It only remains then to note the reading on the tangent galvanometer, and to measure the resistance of the partial circuit CLD . Denoting this resistance by R , the galvanometer deflection by θ , and the electromotive force of A by E , these quantities are connected by the relation

$$E = KR \tan \theta.$$

As K , R , and θ can be readily determined with precision, the method affords a means of making very accurate absolute determinations.

The method is exceedingly simple, and should any errors arise from a falling off in the intensity of the current from B , these may be corrected by closing the circuit only for short intervals at a time during the test.

Since the constant K is given by

$$K = \frac{Ha}{2 n \pi},$$

it is evident that the method may be applied to finding H , the horizontal component of the earth's magnetic action, when the laboratory is provided with a cell whose electromotive force is accurately known.

XVII. ABSOLUTE DETERMINATION OF RESISTANCE BY USE OF A CALORIMETER.

When an electrical current is established in a conductor it is produced and maintained by an expenditure of energy, mechanical, thermal, or chemical, and consequently it may itself

be considered to be a form of energy which is distributed along the conductor, and which reappears in the form of motion, chemical separation, or heat.

If the conditions to which the conductor is subjected are such that the transformation cannot take place into the first two of these forms, the electrical energy will be expended in heat-

ing the conductor, and the amount of heat so produced will be an exact equivalent of the work done in producing the current.

The conductor may then be viewed as a series of points between which, while a current is passing, a certain amount of energy is being expended, and the potential difference between any two points will represent the work done between them in a unit of time when the electric current is of unit intensity.

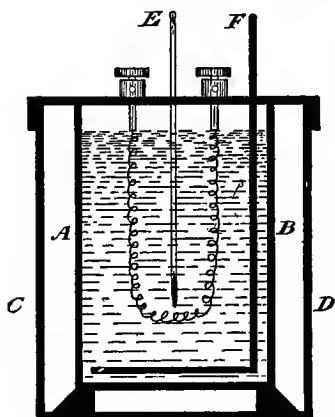


Fig. 50.

If, then, V_1 and V_2 represent in volts the potentials of the two terminals of a conductor forming part of a circuit, and C denotes in amperes the intensity of the current passing between them, $C(V_1 - V_2)T$ watts will represent the energy transformed into heat in the conductor in T seconds.

Denoting by W the amount of this energy in ergs, we have, $W = C(V_1 - V_2)T \cdot 10^7$ ergs, and combining this again with the relation $C = \frac{V_1 - V_2}{R}$, where R is the resistance of the conductor in ohms, we have,

$$W = C^2 R T \cdot 10^7 \text{ ergs.}$$

The mechanical equivalent of heat which is the number of units of work required to be expended to heat one gram of water through one degree centigrade has been found, by experiment, to be about 4.16×10^7 ergs.

Adopting this value, the number of calories or units of heat developed in the conductor is given, therefore, by

$$H = \frac{C^2 \cdot R \cdot T \cdot 10^7}{4 \cdot 16 \cdot 10^7} \text{ calories,}$$

or
$$H = C^2 R T \times .24 \text{ calories,}$$

and the resistance of the conductor may be found in terms of H , T , and C from the relation

$$R = \frac{H}{C^2 T} \times 4.16.$$

The amount of heat developed in the conductor is ascertained by immersing it in a known mass of water, and by noting the gradual rise in the temperature of the latter when the current is passed through it. The apparatus is similar to that shewn in Fig. 50. The water is contained in the vessel AB , which is made of sheet brass, and in order to reduce the errors due to radiation and absorption, this is itself placed on wooden supports in an outer vessel CD , made of the same material. The conductor, which is coiled into a spiral, is attached to two insulated binding poles inserted in the cover of the calorimeter, and is so arranged that when the cover is in position it is completely immersed in the water, and yet does not come in contact with the sides or bottom of AB . The apparatus is also provided with a delicate thermometer E and a stirrer F .

In conducting the experiment the weight of the stirrer and the vessel AB is first determined in grams, and then a known mass of water is placed in it, and the whole lowered into the outer vessel CD . The cover, with coil attached, is then placed in position, and the thermometer is inserted in the opening prepared for it.

The current which is passed through the conductor may be generated by a powerful battery, or, better still, by a dynamo, and the circuit should contain either a sensitive galvanometer,

or a voltameter and a galvanometer, for determining its intensity, as well as a rheostat for modifying it if desired.

When the initial temperature t_1° C. of the water has been accurately ascertained, the current is then turned on, and at the same instant the indication on a stop watch, or other delicate time recorder, is noted.

By means of the rheostat the current is kept constant, and at intervals, as the experiment goes on, both the time and the corresponding temperature of the water is noted, care being taken to keep the latter well stirred.

The temperature of the water should always be initially below that of the room, and the current should be allowed to pass until its temperature has risen as much above that of the room as it was at first below it.

Denoting the final temperature by t_2° C. the time occupied by the experiment by T , and the specific heats of the vessel AB , and the conductor by s_1 and s_2 , we have, if m_2 grams is the mass of the conductor, and m_1 and m those of the vessel, AB and the water in it respectively,

$$H = (m + m_1s_1 + m_2s_2)(t_2 - t_1), \quad (1)$$

and if the intensity of the current is denoted by C , the resistance of the conductor may therefore be found from the relation

$$R = 4.16 \times \frac{(m + m_1s_1 + m_2s_2)(t_2 - t_1)}{C^2T}.$$

The uncertainty which prevails as to the exact value of the mechanical equivalent of heat rather unfits the method for very accurate determinations, and as besides the resistance of a conductor varies with its temperature, the values obtained can only be considered approximations. The method is, however, instructive, and involves an excellent training in the use of the fundamental units of electrical measurements.

A slight modification of the experiment is to determine the resistance offered by the carbon filament of an incandescent

lamp to the passage of a current. This may be accomplished by covering the socket of the lamp with paraffine wax, and noting the heat evolved when it is immersed in the water in the calorimeter. The results obtained illustrate very clearly that the resistance of a carbon conductor rapidly decreases as its temperature increases.

APPENDIX A.

DETERMINATION OF GRAVITY BY THE PENDULUM.

1. If a pendulum of any form be allowed to make small oscillations under the action of gravity, we have the time of a complete oscillation given by the relation $t = 2\pi\sqrt{\frac{l}{g}}$, where l is the length of the equivalent simple pendulum and equal to $\frac{h^2 + k^2}{h}$. If, now, t be observed by means of a clock, and h and k found, we have the value of g . This method is one of the most accurate known for finding the intensity of the earth's attraction at different points on its surface. Various forms have been given to these pendulums from time to time, in order to insure accuracy of measurement, and the most important of those which have been used for the scientific determination of gravity are described below :

(a) *Borda's pendulum.*

Borda (1792) constructed his pendulum so as to realize as nearly as possible the simple pendulum. It was made of a sphere of known radius (a). To render it very heavy it was composed of platinum, and was suspended by a very fine wire about one meter in length.

The knife edge which carried the wire and sphere was so arranged by means of a movable screw as to oscillate in the same time as the complete pendulum. The time was determined by the method of *coincidences*, and g was found from the relation

$$t = 2\pi\sqrt{\frac{l + \frac{2}{5}\frac{a^2}{l}}{g}} \left(1 + \frac{a^2}{16l}\right),$$

where l is the length from the knife edge to the center of the sphere, a the radius of the sphere, and a half the angle of a single oscillation.

(b) *Kater's pendulum.*

In 1818, Captain Kater determined the value of gravity at London by applying to the pendulum the principle discovered by *Huyghens*, that the centers of suspension and of oscillation are reversible.

He made a pendulum of a bar of brass about an inch and a half wide and an eighth of an inch in thickness. This bar was pierced in two places, and triangular knife edges of hard steel were inserted, so that the distance between them was nearly 39 inches.

A large mass in the form of a cylinder was placed near one of the knife edges, being slid on by means of a rectangular opening cut in it; a smaller mass was also attached to the pendulum in such a way as to admit of small motions either way. The pendulum was then swung about the two axes, and adjustment of the masses made until the time of small oscillations was the same. This time being noted, and the distance between the knife edges being accurately measured, g was readily calculated. A small difference being generally found in the two times, it can be shewn that the length of the seconds pendulum is found from the expression,

$$\frac{(h_1 + h_2)(h_1 - h_2)}{t_1^2 h_1 - t_2^2 h_2},$$

where h_1, h_2 are the distances of the center of gravity from the two knife edges, and t_1, t_2 the corresponding times of oscillation.

(c) *Repsold's pendulum.*

It was noticed in experimenting with pendulums made like Kater's, that the vibration is differently affected by the surrounding air according as the large mass is above or below. This led to the form known as *Repsold's*, in which the two ends are exactly similar externally, but the pendulum (which is cylindrical) is hollow at one end. The center of gravity of the figure is equidistant from the knife edges, but the true center of gravity of the whole mass is at a different point.

2. Many observers have, during the present century, conducted observations at different points on the earth's surface in order to determine not only the length of the seconds pendulum but also the excentricity of the earth considered as a spheroid.

Helmert in his *Geodesy* has collected the results of nearly all the more important expeditions, and the following table gives some of the principal stations, with the corresponding lengths of the seconds pendulum and the name of the observer. To find g from this table for any place, the relation

$$\log g = 2 \log \pi + \log l$$

may be used, where l is the length of the seconds pendulum in *centimeters*. The places are arranged geographically in order of their latitudes, and shew thereby the gradual increase in the length of the seconds pendulum as we go from the Equator to the Pole.

PLACE.	LATITUDE.	l .	OBSERVER.
Rawak	0° 1' S.	99.0966	Freycinet.
St. Thomas	0 24 N.	99.1134	Sabine.
Galapagos	0 32 N.	99.1019	Hall.
Para	1 27 S.	99.0948	Foster.
Ascension	7 55 S.	99.1217	—
Sierra Leone	8 29 N.	99.1104	Sabine.
Trinidad	10 38 N.	99.1091	—
Aden	12 46 N.	99.1227	Basevi and Heaviside.
Madras	13 4 N.	99.1168	Basevi and Heaviside.
St. Helena	15 56 S.	99.1581	—
Jamaica	17 56 N.	99.1497	Sabine.
Calcutta	22 32 N.	99.1712	Basevi and Heaviside.
Rio Janeiro	22 55 S.	99.1712	—
Valparaiso	33 2 S.	99.2500	Lütke.
Montevideo	34 54 S.	99.2641	Foster.
Lipari	38 28 N.	99.3097	Biot.
Hoboken, N. J.	40 44 N.	99.3191	—
Tiflis	41 41 N.	99.3190	—
Toulon	43 7 N.	99.3402	Duperrey.
Bordeaux	44 50 N.	99.3470	Biot.
Padua	45 24 N.	99.3623	Biot.
Paris	48 40 N.	99.3858	—
Shanklin Farm (Isle of Wight) .	50 37 N.	99.4042	Kater.
Kew	51 28 N.	99.4169	—
Greenwich	51 28 N.	99.4143	—
London	51 31 N.	99.4140	—
Berlin	52 30 N.	99.4235	—
Staten Island	54 46 S.	99.4501	Foster.
Cape Horn	55 51 S.	99.4565	Foster.
Leith	55 58 N.	99.4550	—
Sitka	57 3 N.	99.4621	Lütke.
Pulkowa	59 46 N.	99.4854	Sawitsch.
Petersburg	59 56 N.	99.4876	—
Unst	60 45 N.	99.4959	—

Those places in the preceding table for which the lengths of the seconds pendulum have been calculated from a number of observations made by different observers are indicated by a dash.

3. During the past few years several observers have made observations on the value of g at different points in North America. Professor Mendenhall, of the U. S. Coast Survey, during the summer of 1891, visited a number of places on the Pacific coast between San Francisco and Alaska, and in his report of the expedition gives a table of the values determined, with the places and corresponding latitudes. He made use of a half-seconds pendulum, inclosed in an air-tight chamber, which could be exhausted with an air-pump. A special method was used for noting the coincidences. (See *U. S. Coast and Geodetic Survey*, Report for 1891, part 2.)

4. Defforges, one of the greatest living authorities on methods of gravity determination, crossed from Washington to San Francisco during the summer of 1893, and made a number of observations, which are given in the following table. The value of g alone is given :

Washington	.	.	.	.	.	.	.	980.169
Montreal	.	.	.	.	.	.	.	980.747
Chicago	.	.	.	.	.	.	.	980.375
Denver	.	.	.	.	.	.	.	980.983
Salt Lake City	.	.	.	.	.	.	.	980.050
Mt. Hamilton	.	.	.	.	.	.	.	979.916
San Francisco	.	.	.	.	.	.	.	980.037

These are all reduced to sea-level.

NOTE.—An excellent book of reference on the subject of gravity determination is *Mémoires relatifs à la Physique*, Tome IV, Paris, 1889.

APPENDIX B.

THE TORSION PENDULUM.

IN many instruments of precision designed for investigations in Electricity, Magnetism, and Gravitation, certain portions of the apparatus, which are movable about a vertical axis, are suspended either by metallic wires or elastic fibers.

The suspension may be either unifilar or bifilar. If the former method is adopted and suspension is all that is required, fibers are selected, such as silk cocoon threads, which offer practically no resistance to torsion, and are capable of supporting a considerable mass. If, however, a couple, or a number of couples, is applied to the movable part of the instrument, and this system is equilibrated and measured by the torsion in the suspending fibers or wires, the latter are generally made from some metal, such as silver, or from fine glass or quartz threads.

In the bifilar suspension the movable apparatus is attached to two threads or fibers of the same length, which have their upper ends fastened to two fixed points. When the suspended system is displaced from its position of rest, it is its weight which gives rise to the directive force, and the restorative couple is but slightly affected by any torsion which may exist in the suspending threads.

The elastic properties of substances used for suspension purposes may be investigated by means of the Torsion Pendulum. In such an apparatus a wire or fiber of the material in question is suspended from some point to which it is firmly attached, and a heavy body called a vibrator is securely fastened to its lower extremity.

Various forms have been suggested for this vibrator. It is generally made of some non-magnetic substance such as brass or copper, and it must be of such dimensions that its moment of inertia about the axis of the suspending wire can be readily calculated. Professor Andrew Gray recommends a hollow circular cylinder of brass or copper, as it is best suited for a correct determination of the moment of inertia, and as besides its oscillations are but slightly affected by the presence of the air.

Professor Threlfall, in his article* on "The Elastic Constants of Quartz Threads," states that the vibrators used by him in his investigations "were (1) a pure silver anchor ring supported by a disc of aluminium foil through the center of which passed a short bit of aluminium wire to which the thread was attached, (2) a cylindrical vibrator of brass, vibration being about the axis of the cylinder, and (3) a disc of brass, vibration taking place about its axis of revolution."

When the vibrator is given a rotation about the axis of the suspending wire or thread, and is then left to itself, it will perform oscillations about its mean position which within the limits of perfect elasticity are isochronous. This at once indicates that the motion is harmonic, and that therefore the moment of the restorative couple is proportional to the angle through which the suspended body is rotated. This moment is also proportional to the fourth power of the diameter of the wire, and varies inversely as its length. As formulated originally by Coulomb these laws are exhibited in the relation

$$c = \mu \cdot \frac{d^4}{l} \cdot \theta, \quad (1)$$

where d is the diameter of the wire, l its length, and θ the angular displacement.

The time of an oscillation is therefore given by

$$t = \frac{2\pi}{d^2} \sqrt{\frac{KI}{\mu}}, \quad (2)$$

K being the moment of inertia of the vibrator about the axis of rotation.

In order to verify these laws, it is only necessary to select a number of wires of different dimensions, but of the same material. If the vibrator be suspended by each of them in turn, and the time of an oscillation observed with each, it will be found that if these values together with the corresponding ones for d , l , and K be substituted in equation (2), a constant value will be obtained for μ .

This quantity, which is termed the *constant*, or *coefficient of torsion*, depends only on the nature of the suspending wire, and on its temperature. In the C.G.S. system of units it is the numerical expression for the couple which, if applied to one base of a circular cylinder of wire, whose length and diameter are each one centimeter, would twist this base through a unit angle, the other base being fixed.

* Phil. Mag., July, 1890.

The following table gives the values of this coefficient for a number of metals, and for silk and quartz fibers :

SUBSTANCE.	COEFFICIENT OF TORSION.	SUBSTANCE.	COEFFICIENT OF TORSION.
Aluminium	2.5516×10^{10}	Copper	4.2458×10^{10}
Silver	2.6144	German Silver	4.7412
Gold	2.6958	Platinum	6.6659
Zinc	3.2559	Iron	7.4448
Brass	3.3727	Quartz (fibers)	2.8289
Platinum-Silver (1 Pt + 2 Ag)	3.5581	* Silk (fibers)	

* Professor T. Gray in 1886 made a thorough investigation of the elastic properties of silk threads, and found that the torsion factor for fibers of Japanese silk one centimeter in length, and of diameters ranging from .0008 to .0015 cms. varied from .00096 to .0025 of a dyne.

Metallic wires have been found quite unsuitable for suspension purposes when delicate manipulation is demanded. Their coefficients of torsion vary considerably with changes in temperature, and are slightly diminished when the wires have been allowed to oscillate for a long period. Permanent deformations also soon appear in them unless the amplitudes of the oscillations are very small, and consequently the angular displacements cannot be relied on in making accurate measurements of applied couples.

The latter difficulty was experienced by Professor C. V. Boys* in his recent investigation on the Newtonian Constant of Gravitation. After experimenting with many substances he found fibers of quartz to possess elastic properties more nearly perfect than those of any other material. Such fibers can be drawn to any required length, and so exceedingly fine that even a powerful microscope fails to reveal their presence. They are, however, very regular, are as strong as steel, and remain constant for a long period. With them the suspended body can be permitted to oscillate through large angles without any deformation or displacement of the zero position appearing. On account of these many excellent properties, they are, therefore, most suitable for suspensions in investigations demanding accuracy and delicacy of manipulation.

When a magnet whose magnetic moment is known is suspended in a field of determined intensity, the coefficient of torsion can also be found

* Phil. Mag., June, 1887.

by observing the deflections of the magnet from its position of rest when the suspending fiber or wire is subjected to different amounts of torsion.

If θ denote the deflection of the magnet corresponding to the torsion in the wire ϕ , and H and M the intensity of the field and the magnetic moment respectively, these quantities are connected by the relation

$$\mu \frac{d^4 \phi}{l} = HM \sin \theta,$$

and the constant of torsion is therefore given by

$$\mu = \frac{HML \sin \theta}{d^4 \phi}. \quad (3)$$

When this method is adopted, care must be taken to have the magnet initially at rest, without the suspending wire being subjected to any torsional strain. Further details of the method have already been given on page 187.

Determination of moments of inertia.

By a reference to equation (2) it can be seen that if the constant of torsion for a given wire μ is known, the moment of inertia K of the suspended body can be at once deduced by finding the time of an oscillation.

It is better, however, owing to the difficulty in obtaining a value for μ sufficiently exact for this purpose, to adopt a comparative method. The body whose moment of inertia is required is first attached to the suspending wire, and its time of oscillation found. It is then replaced by some body whose moment of inertia is known or can be readily calculated, and the oscillation period is again determined.

Since the directive couple in the two cases is the same, the constant of torsion can then be eliminated. If t_1 and t are the two periods of oscillation, and K_1 and K the required moment of inertia and that which is known respectively, it follows from equation (2) that

$$K_1 = K \frac{t_1^2}{t^2}.$$

A modification of this method is to use a bar magnet for the auxiliary body, and to determine the vibration period first with the given body attached, and then with it removed. The restorative couple will in this case depend on the intensity of the magnetic field as well as on the coefficient of torsion of the wire. The vibrations must also be taken of small amplitude.

TABLES.

TABLE I.

$$\pi = 3.1416, \quad \frac{1}{\pi} = .3183,$$

$$\pi^2 = 9.8696, \quad \sqrt{\pi} = 1.7724.$$

Circle, diameter $2a$:

$$\begin{aligned} \text{circumference} &= 2\pi a, \\ \text{area} &= \pi a^2. \end{aligned}$$

Ellipse, axes $2a$, $2b$:

$$\text{area} = \pi ab.$$

Triangle, base b , altitude a :

$$\text{area} = \frac{1}{2} ab.$$

Sphere, radius a :

$$\begin{aligned} \text{surface} &= 4\pi a^2, \\ \text{volume} &= \frac{4}{3}\pi a^3. \end{aligned}$$

Right circular cylinder, radius a , length l :

$$\begin{aligned} \text{surface (including ends)} &= 2\pi a(l + a), \\ \text{volume} &= \pi a^2 l. \end{aligned}$$

Right circular cone, radius of base b , altitude a :

$$\begin{aligned} \text{surface} &= \pi b(\sqrt{a^2 + b^2} + b), \\ \text{volume} &= \frac{1}{3}\pi ab^2. \end{aligned}$$

TABLE II.

1 meter	= 100 centimeters = 1000 millimeters.
1 liter	= 1000 cubic centimeters.
1 kilogram	= 1000 grams.
1 gram	= 10 decigrams = 100 centigrams = 1000 milligrams.

1 meter (1 m.) = 39.371 inches.
 1 inch = 25.4 millimeters.
 1 millimeter (1 mm.) = .039371 inches.

1 cubic centimeter of water at 4°C. = 1 gram.

1 cubic inch = 16.386 cubic centimeters (cc.).
 1 pint = 567.93 cubic centimeters.
 1 liter = 1.76 pints.

1 ounce avoirdupois = 28.35 grams.

1 pound avoirdupois = 453.593 grams.

1 gram = 15.432 grains.

1 kilogram = 2.2046 pounds.

TABLE III.

Specific Gravities of Solids.

Aluminium	2.6	Ice	.918
Amber	1.1	Iceland spar	2.7
Antimony	6.7	India rubber	.99
Bismuth	9.8	Iron (cast)	7.2
Bone	1.9	Iron (wrought)	7.79
Brass }	8.4	Iron (steel)	7.79
Bronze }		Ivory	1.92
Carbon (gas)	1.8	Lead	11.4
Copper	8.95	Lignum vitæ	1.3
Cork	.24	Nickel	8.57
Diamond	3.5	Platinum	21.5
Ebony	1.19	Sand	1.42
German silver	8.62	Silver (925 fine)	10.38
Glass (green)	2.64	Silver (pure)	10.57
Glass (crown)	2.5	Starch	1.53
Glass (flint)	3.3	Sugar (cane)	1.6
Gold (18 carat)	14.88	Salt	.92
Gold (pure)	19.3	Tin	7.3
Graphite	2.2	Wax (bees')	.96
Gunpowder	2.03	Zinc (rolled)	7.2
Guttapercha	.97	Zinc (cast)	6.86
Human body	1.07		

Specific Gravities of Liquids.

Alcohol (ethyl) at 15° C.	.79
Ammonia at 15° C.	.761
Benzine at 15° C.	.89
Bisulphide of carbon at 15° C.	1.28
Chloroform at 15° C.	1.5
Ether at 15° C.	.72
Glycerine at 15 C.	1.26
Hydrochloric acid at 15° C.	1.2
Mercury at zero C.	13.596
Milk (cows') at zero C.	1.03
Nitric acid at 15° C.	1.52
Olive oil at 15° C.	.915
Sea water at 0° C.	1.026
Turpentine at 15° C.	.872
Water at 4° C.	1.

Specific Gravities of Gases referred to Air at 0° C. and 760 mm.

Air	1.
Oxygen	1.10563
Nitrogen	.97137
Hydrogen	.06926
Carbon dioxide	1.52901

Vapours.

Alcohol (ethyl)	1.61 at 78°.4 C.
Chloroform	4.20 at 60°.8
Water	.64 at 100°
Ether	2.59 at 35°.5
Iodine	8.72 at 175°
Mercury	6.98 at 350°

1 liter of dry air at 0° C. and 760 mm. pressure weighs 1.293 grams.

TABLE IV.

Specific Heats of Solids and Liquids referred to Water.

Aluminium	.202		
Antimony	.0507		
Bismuth	.0305		
Brass	.094		
Copper	.095		
Glass	.2		
Gold	.0324		
Graphite	.36		
Ice	.5		
Iron	.112		
Lead	.0315		
Platinum	.0356		
Quartz	.19		
Silver	.0559		
Sulphur	.184		
Tin	.0559		
Zinc	.0935		

		<i>Liquids at 15° C.</i>	
		Alcohol	.615
		Chloroform	.233
		Ether	.517
		Mercury	.033

		<i>Gases at Constant Pressure.</i>	
		Air	.2375
		Steam	.4805

TABLE V.

Latent Heats of Fusion.

Beeswax	97.22
Lead	5.37
Steam	536.
Sulphur	9.35
Water	79.25
Zinc	28.15

TABLE VI.

Cubical Expansion of Solids and Liquids for 1° C.

Copper	.00005
Glass	.000023-28
Iron	.0000355
Platinum	.000026
Silver	.0000583
Tin	.000069
Zinc	.000089

Liquids (Mean Expansion).

Alcohol	.00108
Chloroform . . .	.0014
Ether	.0021
Mercury	.0001815
Turpentine . . .	.00105
Water	.00012 at 15° C. to .00074 at 90° C.

Gases expand .003665 of their volume for each degree Centigrade.

TABLE VII.

*Expansion of Water from 0° to 100°.**Volume of 1 Gram of Water in Cubic Centimeters.*

TEMPERATURE.	VOLUME.	INCREASE PER 1°.
0°	1.0001	
4	1.0000	
10	1.0003	
15	1.0009	.00012
20	1.0017	.00016
25	1.0029	.00024
30	1.0043	.00028
35	1.0059	.00032
40	1.0077	.00036
45	1.0097	.00040
50	1.0120	.00046
55	1.0144	.00048
60	1.0170	.00052
65	1.0197	.00054
70	1.0227	.00060
75	1.0258	.00062
80	1.0290	.00064
85	1.0323	.00066
90	1.0358	.00070
95	1.0395	.00074
100	1.0432	.00074

TABLE VIII.

*Boiling Temperature of Water (t) at Barometer Pressure (b)
(after Regnault).*

b	t	b	t	b	t	b	t	b	t
680	96°.92	700	97°.72	720	98°.49	740	99°.26	760	100°.00
681	.96	701	.75	721	.53	741	.29	761	.04
682	97.00	702	.79	722	.57	742	.33	762	.07
683	.04	703	.83	723	.61	743	.37	763	.11
684	.08	704	.87	724	.65	744	.41	764	.15
685	.12	705	.91	725	.69	745	.44	765	.18
686	.16	706	.95	726	.72	746	.48	766	.22
687	.20	707	.99	727	.76	747	.52	767	.26
688	.24	708	98.03	728	.80	748	.56	768	.29
689	.28	709	.07	729	.84	749	.59	769	.33
690	.32	710	.11	730	.88	750	.63	770	.36
691	.36	711	.15	731	.92	751	.67	771	.40
692	.40	712	.19	732	.95	752	.70	772	.44
693	.44	713	.22	733	.99	753	.74	773	.47
694	.48	714	.26	734	99.03	754	.78	774	.51
695	.52	715	.30	735	.07	755	.82	775	.55
696	.56	716	.34	736	.11	756	.85	776	.58
697	.60	717	.38	737	.14	757	.89	777	.62
698	.64	718	.42	738	.18	758	.93	778	.65
699	.68	719	.46	739	.22	759	.96	779	.69
700	97°.72	720	98°.49	740	99°.26	760	100°.00	780	100°.72

TABLE IX.

For Hygrometry.

Pressure of aqueous vapour e and weight of water f contained in 1 cubic meter of air with dew-point t ; or, when at the temperature t , the air would be saturated with aqueous vapour.

t	e	f	t	e	f	t	e	f	t	e	f
	mm.	gr.		mm.	gr.		mm.	gr.		mm.	gr.
-10°	2.0	2.1	0°	4.6	4.9	10°	9.1	9.4	20	17.4	17.2
- 9	2.2	2.4	1	4.9	5.2	11	9.8	10.0	21	18.5	18.2
- 8	2.4	2.7	2	5.3	5.6	12	10.4	10.6	22	19.7	19.3
- 7	2.6	3.0	3	5.7	6.0	13	11.1	11.3	23	20.9	20.4
- 6	2.8	3.2	4	6.1	6.4	14	11.9	12.0	24	22.2	21.5
- 5	3.1	3.5	5	6.5	6.8	15	12.7	12.8	25	23.6	22.9
- 4	3.3	3.8	6	7.0	7.3	16	13.5	13.6	26	25.0	24.2
- 3	3.6	4.1	7	7.5	7.7	17	14.4	14.5	27	26.5	25.6
- 2	3.9	4.4	8	8.0	8.1	18	15.4	15.1	28	28.1	27.0
- 1	4.2	4.6	9	8.5	8.8	19	16.3	16.2	29	29.8	28.6
0°	4.6	4.9	10	9.1	9.4	20	17.4	17.2	30	31.6	30.1

TABLE X.

SCALE OF PHYSICISTS.			SCALE OF EQUAL TEMPERAMENT.	
ut	256	512	ut	256
re	288	576	ut [#]	271.22
mi	320	640	re	287.35
fa	341.33	682.66	re [#]	304.437
sol	384	768	mi	322.539
la	426.66	853.33	fa	341.719
si	480	960	fa [#]	362.038
			sol	383.566
			sol [#]	406.374
			la	430.539
			la [#]	456.140
			si	483.263

TABLE XI.

Mean Indices of Refraction, and Dispersions of Several Bodies.

	INDEX OF REFRACTION.	DISPERSION.
Crown glass (mean)	1.53	.022
Flint glass	1.60	.042
Water	1.336	.0132
Alcohol	1.372	.0133
Carbon disulphide	1.68	.0837
Canada balsam	1.54	
Air	1.000294 . .	

TABLE XII A.

*Elements of Terrestrial Magnetism at Toronto.*Latitude, $43^{\circ} 39' 36''$. Longitude, 5 h. 17 m. 34.65 s.

DATE.	DECLINATION.	INCLINATION.	HORIZONTAL FORCE IN C.G.S. UNITS.
	(westerly)		
1885	$3^{\circ} 59'.8$	$74^{\circ} 51'.6$	.165579
1886	$4^{\circ} 2'.1$	$74^{\circ} 48'.9$	.165717
1887	$4^{\circ} 4'.8$	$74^{\circ} 47'.6$	.165875
1888	$4^{\circ} 8'.3$	$74^{\circ} 46'.5$	.165993
1889	$4^{\circ} 12'.0$	$74^{\circ} 44'.7$	.166111
1890	$4^{\circ} 18'.2$	$74^{\circ} 42'.2$	.166150
1891	$4^{\circ} 23'.3$	$74^{\circ} 37'.5$	.166170
1892	$4^{\circ} 29'.2$	$74^{\circ} 37'.2$	.166229
1893	$4^{\circ} 36'.4$	$74^{\circ} 36'.2$	.166354
1894	$4^{\circ} 42'.3$	$74^{\circ} 34'.7$	.166334

TABLE XII B.

Elements of Terrestrial Magnetism at Other Places.

DATE.	PLACE.	DECLINATION.	INCLINATION.	HORIZONTAL FORCE IN C.G.S. UNITS.
		(westerly)		
1887	Greenwich	$17^{\circ} 49'.1$	$67^{\circ} 26' 26''$	.18175
1888	"	$17^{\circ} 40'.4$	$67^{\circ} 25' 25''$	.18204
1889	"	$17^{\circ} 34'.9$	$67^{\circ} 24' 9''$	.18201
1890	"	$17^{\circ} 28'.6$	$67^{\circ} 22' 52''$	.18232
1891	"	$17^{\circ} 23'.4$	$67^{\circ} 21' 24''$	.18254
1889	Washington	$4^{\circ} 1' 31''$	$71^{\circ} 5' 59''$	.198693
1890	"	$4^{\circ} 5' 45''$	$71^{\circ} 4' 31''$	.198604
1891	"	$4^{\circ} 9' 43''$	$71^{\circ} 5' 4''$	.198550
1892	"	$4^{\circ} 14' 12''$	$71^{\circ} 3' 55''$	.198485
1886	Paris	$16^{\circ} 00'.9$	$65^{\circ} 15'.8$	.19439
1887	"	$15^{\circ} 54'.8$	$65^{\circ} 14'.7$	.19470
1888	"	$15^{\circ} 49'.7$	$65^{\circ} 14'.5$	.19496
1889	"	$15^{\circ} 44'.6$	$65^{\circ} 12'.6$	.19522
1890	"	$15^{\circ} 38'.7$	$65^{\circ} 11'.$	.19543
1891	"	$15^{\circ} 32'.8$	$65^{\circ} 10'.1$	.19558

TABLE XIII.

Cross-Section of Round Wires, with Resistance, Conductivity, and Weight of Pure Copper Wires, according to the Birmingham Wire Gauge.

TEMPERATURE 15° C.

B. W.G.	DIAMETER.		AREA OF CROSS-SECTION.		RESISTANCE.		CONDUCTIVITY.		WEIGHT (Density = 8.95).	
	Ins.	Cms.	Sq. Ins.	Sq. Cms.	Legal Ohms per Yard.	Legal Ohms per Meter.	Yards per Legal Ohm.	Meters per Legal Ohm.	Lbs. per Yard.	Grams per Meter.
.0000	.454	1.153	.162	1.0444	.000150	.000165	6640	6072	1.884	934.7
.000	.425	1.079	.142	.915	.000172	.000188	5819	5321	1.651	819.1
.00	.380	.965	.113	.732	.000215	.000235	4653	4254	1.320	654.8
0	.340	.864	.0908	.586	.000269	.000294	3735	3415	1.056	524.2
1	.300	.762	.0707	.456	.000345	.000378	2899	2652	.822	408.1
2	.284	.721	.0633	.409	.000385	.000420	2599	2377	.737	365.8
3	.259	.658	.0527	.340	.000463	.000506	2162	1996	.613	304.2
4	.238	.605	.0445	.287	.000548	.000599	1825	1669	.518	256.9
5	.220	.559	.0380	.245	.000642	.000701	1561	1427	.442	219.5
6	.203	.516	.0324	.209	.000754	.000824	1328	1214	.377	186.9
7	.180	.457	.0254	.164	.000958	.00105	1044	1004	.296	146.9
8	.165	.419	.0214	.138	.00114	.00125	877	802	.249	123.5
9	.148	.376	.0172	.111	.00141	.00155	706	645	.200	99.3
10	.134	.340	.0141	.0910	.00173	.00189	578	529	.164	81.4
11	.120	.305	.0113	.0730	.00216	.00235	463	424	.132	65.5
12	.109	.277	.00933	.0602	.00261	.00286	382	350	.109	53.9
13	.095	.241	.00709	.0457	.00344	.00376	291	266	.0825	40.9
14	.083	.211	.00541	.0349	.00451	.00492	221	203	.0630	31.2
15	.072	.183	.00407	.0263	.00599	.00655	167	153	.0474	23.5
16	.065	.165	.00331	.0214	.00735	.00804	136	124	.0386	19.2
17	.058	.147	.00264	.0170	.00923	.0101	108	98.7	.0307	15.3
18	.049	.124	.00189	.0122	.0130	.0141	77.3	70.7	.0220	10.9
19	.042	.107	.00139	.00894	.0176	.0194	57.8	52.0	.0161	8.00
20	.035	.0889	.000962	.00621	.0253	.0277	39.4	36.1	.0122	5.56
21	.032	.0813	.000804	.00519	.0304	.0331	32.0	30.1	.00936	4.64
22	.028	.0711	.000616	.00397	.0395	.0433	25.3	23.1	.00716	3.55
23	.025	.0635	.000491	.00317	.0406	.0543	20.2	18.4	.00571	2.83
24	.022	.0559	.000380	.00245	.0642	.0701	15.6	14.3	.00442	2.19
25	.020	.0508	.000314	.00203	.0778	.0849	12.8	11.7	.00307	1.82
26	.018	.0457	.000254	.00164	.0959	.105	10.2	9.53	.00266	1.47
27	.016	.0406	.000201	.00130	.122	.133	8.25	7.54	.00234	1.16
28	.014	.0356	.000154	.000993	.158	.173	6.31	5.77	.00179	.889
29	.013	.0330	.000133	.000856	.184	.201	5.41	4.98	.00154	.766
30	.012	.0305	.000113	.000732	.216	.235	4.64	4.24	.00132	.653
31	.010	.0254	.0000785	.000507	.311	.339	3.23	2.95	.000915	.454
32	.009	.0229	.0000636	.000410	.384	.419	2.51	2.39	.000746	.367
33	.008	.0203	.0000503	.000324	.486	.530	2.06	1.88	.000585	.290
34	.007	.0178	.0000385	.000248	.634	.693	1.58	1.45	.000442	.220
35	.005	.0127	.0000196	.000127	1.25	1.35	.806	.736	.000229	.113
36	.004	.0102	.0000126	.0000811	1.94	2.13	.516	.471	.000146	.0726

TABLE XIV.

Cross-Section of Round Wires, with Resistance, Conductivity, and Weight of Hard-Drawn Pure Copper Wires, according to the New Standard Wire Gauge (Legalized Aug. 23, 1883, Great Britain and Ireland).

TEMPERATURE 15° C.

Descriptive No.	DIAMETER.		AREA OF CROSS-SECTION.		RESISTANCE.		CONDUCTIVITY.		WEIGHT (Density = 8.95).	
	Ins.	Cms.	Sq. Ins.	Sq. Cms.	Legal Ohms per Yard.	Legal Ohms per Meter.	Yards per Legal Ohm.	Meters per Legal Ohm.	Lbs. per Yard.	Grams per Meter.
0000000	.500	1.270	.1963	1.267	.000125	.000136	8055	7365	2.285	1134
0000000	.464	1.179	.1690	1.091	.000144	.000157	6937	6343	1.970	976.3
0000000	.432	1.097	.1466	.946	.000166	.000182	6013	5498	1.706	846.3
0000000	.400	1.016	.1257	.811	.000194	.000213	5054	4714	1.463	725.6
0000000	.372	.945	.1087	.701	.000225	.000245	4459	4077	1.265	627.6
0000000	.348	.884	.0951	.614	.000256	.000280	3901	3568	1.107	549.6
0000000	.324	.823	.0824	.532	.000296	.000323	3384	3093	.960	476.1
0000000	.300	.762	.0707	.456	.000345	.000377	2899	2652	.823	408.1
0000000	.276	.701	.0598	.386	.000408	.000446	2454	2244	.696	345.4
0000000	.252	.640	.0499	.322	.000489	.000536	2046	1871	.581	288.0
0000000	.232	.589	.0423	.273	.000577	.000631	1734	1586	.492	244.1
0000000	.212	.538	.0353	.228	.000691	.000756	1451	1324	.411	203.8
0000000	.192	.488	.0290	.187	.000842	.000921	1197	1086	.337	166.8
0000000	.176	.447	.0243	.157	.00100	.00110	988	912	.283	140.5
0000000	.160	.406	.0201	.130	.00122	.00135	824	748	.234	116.1
0000000	.144	.366	.0163	.105	.00149	.00164	660	611	.190	94.0
0000000	.128	.325	.0129	.0830	.00190	.00208	528	482	.150	74.3
0000000	.116	.295	.0106	.0682	.00230	.00252	434	396	.123	61.0
0000000	.104	.264	.00849	.0548	.00287	.00314	348	318	.0989	49.0
0000000	.092	.234	.00665	.0429	.00367	.00402	273	250	.0774	38.4
0000000	.080	.203	.00503	.0324	.00485	.00530	206	188	.0585	29.0
0000000	.072	.183	.00407	.0263	.00659	.00717	167	153	.0474	23.5
0000000	.064	.163	.00322	.0208	.00975	.00839	132	120	.0374	18.6
0000000	.056	.142	.00246	.0159	.0099	.0108	101	91.5	.0287	14.2
0000000	.048	.122	.00181	.0117	.0135	.0147	74.2	67.8	.0211	10.4
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0000000	.0136	.0345	.000145	.000937	.168	.183	5.95	5.45	.00169	.839
0000000	.0124	.0315	.000121	.000779	.202	.221	4.86	4.53	.00141	.697
0000000	.0116	.0295	.000106	.000682	.230	.252	4.34	3.96	.00123	.610
0000000	.0108	.0274	.0000916	.000591	.266	.291	3.75	3.44	.00107	.529
0000000	.0100	.0254	.0000785	.000507	.311	.339	3.22	2.94	.000914	.453
0000000	.0092	.0234	.0000665	.000429	.367	.402	2.73	2.50	.000774	.384
0000000	.0084	.0213	.0000554	.000358	.440	.481	2.27	2.08	.000645	.320
0000000	.0076	.0193	.0000454	.000293	.540	.587	1.86	1.70	.000548	.262
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0000000	.0048	.0122	.0000181	.000117	1.32	1.47	.759	.679	.000211	.104
0000000	.0044	.0112	.0000152	.0000981	1.60	1.75	.624	.570	.000177	.0878
0000000	.0040	.0102	.0000126	.0000811	1.94	2.13	.516	.471	.000146	.0726
0000000	.0036	.00914	.0000102	.0000657	2.39	2.62	.418	.382	.000118	.0588
0000000	.0032	.00813	.00000804	.0000519	3.04	3.32	.330	.301	.0000936	.0464
0000000	.0028	.00711	.00000616	.0000397	3.96	4.33	.253	.230	.0000717	.0356
0000000	.0024	.00610	.00000452	.0000292	5.39	5.90	.185	.170	.0000527	.0261
0000000	.0020	.00508	.00000314	.0000203	7.76	8.49	.128	.118	.0000366	.0181
0000000	.0016	.00406	.00000201	.0000130	12.20	13.3	.0824	.0754	.0000234	.0116
0000000	.0012	.00305	.00000113	.00000730	21.6	23.5	.0464	.0425	.0000132	.00653
0000000	.0010	.00254	.00000085	.00000507	31.1	33.9	.0322	.0294	.00000914	.00453

TABLE XV.

Specific Resistances of Wires of Different Metals and Alloys.

NAME OF CONDUCTOR.	SPECIFIC RESISTANCE IN OHMS.	RESISTANCE IN OHMS.	
		1 METER WEIGHING 1 GRAM.	100 METERS 1 MM. IN DIAMETER.
Silver (annealed) . . .	1.492×10^{-6}	.1517	1.899
Silver (hard drawn) . . .	1.62	.165	2.062
Copper (annealed) . . .	1.584	.1415	2.017
Copper (hard drawn) . . .	1.621	.1443	2.063
Gold (annealed)	2.041	.4007	2.598
Gold (hard drawn) . . .	2.077	.4076	2.644
Aluminium (annealed) . .	2.889	.0743	3.678
Zinc (compressed) . . .	5.58	.3995	7.105
Platinum (annealed) . . .	8.981	1.925	11.435
Iron (annealed)	9.636	.7518	12.27
Nickel (annealed)	12.356	1.052	15.73
Tin (annealed)	13.103	.9564	16.68
Lead (compressed)	19.465	2.217	24.78
Antimony (compressed) . .	35.21	2.37	44.83
Bismuth (compressed) . . .	130.1	12.8	165.60
Mercury (liquid *)	94.34	12.826	120.11
Alloy (2 Pt + 1 Ag) . . .	24.187	2.907	30.79
Alloy (2 Au + 1 Ag) . . .	10.776	1.638	13.72
Alloy (9 Pt + 1 Ir)	21.633	4.651	27.54
German silver	20.76	1.817	26.43

* According to a very careful determination by Lord Rayleigh and Mrs. Sidgwick (*Phil. Trans.*, Part I, 1883), a column of mercury, one square millimeter in cross-section, which at 0°C . has a resistance of an ohm, is 106.21 centimeters in length. At an International Conference at Paris in 1884 it was agreed to define the "legal ohm" as "the resistance of a column of mercury 106 centimeters long and one square millimeter in section at the temperature of melting ice." The Chamber of Delegates at the Chicago Electrical Congress in 1893 adopted "as a unit of resistance the international ohm, which is based upon the ohm equal to 10^9 units of resistance of the C.G.S. system of electromagnetic units, and is represented sufficiently well by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area, and of a length of 106.3 centimeters.

TABLE XVI.

*Conductivities of Pure Metals at t° C.***Conductivity at $0^{\circ} = 1$.*

MEtal.	CONDUCTIVITY AT t° C.
Silver	$1 - .0038278 t + .000009848 t^2$
Copper	$1 - .0038701 t + .000009009 t^2$
Gold	$1 - .0036745 t + .000008443 t^2$
Zinc	$1 - .0037047 t + .000008274 t^2$
Cadmium	$1 - .0036871 t + .000007575 t^2$
Tin	$1 - .0036029 t + .000006136 t^2$
Lead	$1 - .0038756 t + .000009146 t^2$
Arsenic	$1 - .0038996 t + .000008879 t^2$
Antimony	$1 - .0030826 t + .000010364 t^2$
Bismuth	$1 - .0035216 t + .000005728 t^2$
Iron	$1 - .0051182 t + .000012916 t^2$

* From Matthiessen.

TABLE XVII.

*Conductivity and Resistance of Pure Copper at Temperatures from
0° C. to 40° C.*

TEMPERATURE.	CONDUCTIVITY.	RESISTANCE.	TEMPERATURE.	CONDUCTIVITY.	RESISTANCE.
0°	1.0000	1.0000	21°	.9227	1.0838
1	.9961	1.00388	22	.9192	1.0879
2	.9923	1.00776	23	.9158	1.0920
3	.9885	1.0116	24	.9123	1.0961
4	.9847	1.0156	25	.9089	1.1003
5	.9809	1.0195	26	.9054	1.1044
6	.9771	1.0234	27	.9020	1.1085
7	.9734	1.0274	28	.8987	1.1127
8	.9696	1.0313	29	.8953	1.1169
9	.9659	1.0353	30	.8920	1.1211
10	.9622	1.0393	31	.8887	1.1253
11	.9585	1.0433	32	.8854	1.1295
12	.9549	1.0473	33	.8821	1.1337
13	.9512	1.0513	34	.8788	1.1379
14	.9476	1.0553	35	.8756	1.1421
15	.9440	1.0593	36	.8723	1.1464
16	.9404	1.0634	37	.8691	1.1506
17	.9368	1.0675	38	.8659	1.1548
18	.9333	1.0715	39	.8628	1.1591
19	.9297	1.0756	40	.8596	1.1633
20	.9262	1.0797			

TABLE XVIII A.

Liquid Resistances.

SUBSTANCE DISSOLVED.	COMPOSITION.	TEMPERATURE.	SPECIFIC RESISTANCE.
Sulphuric acid	SO_3HO	15° C.	9.146 ohms
	$\text{SO}_3\text{HO} + 14 \text{ HO}$	19° C.	1.336 "
	$\text{SO}_3\text{HO} + 13 \text{ HO}$	22° C.	1.256 "
	$\text{SO}_3\text{HO} + 499 \text{ HO}$	22° C.	17.431 "
Sulphate of zinc . . .	$\text{ZnOSO}_3 + 23 \text{ HO}$	23° C.	18.31 "
	$\text{ZnOSO}_3 + 24 \text{ HO}$	23° C.	18.02 "
	$\text{ZnOSO}_3 + 105 \text{ HO}$	23° C.	33.04 "
Sulphate of copper . .	$\text{CuOSO}_3 + 45 \text{ HO}$	22° C.	19.10 "
	$\text{CuOSO}_3 + 105 \text{ HO}$	12° C.	31.42 "
Sulphate of magnesia	$\text{MgOSO}_3 + 34 \text{ HO}$	22° C.	18.44 "
	$\text{MgOSO}_3 + 107 \text{ HO}$	22° C.	30.06 "
Hydrochloric acid . .	$\text{HCl} + 7.5 \text{ HO}$	23° C.	1.285 "
	$\text{HCl} + 250 \text{ HO}$	23° C.	8.177 "

TABLE XVIII B.

*Specific Resistances of Sulphuric Acid at 22° C.**(Kohlrausch and Nippoldt.)*

DENSITY OF THE SOLUTION.	PROPORTION OF ACID.	SPECIFIC RESISTANCE.	RELATIVE INCREASE OF CONDUCTIVITY FOR 1°C.
.9985	.0	70.41	.47.10 ⁻²
1.0000	.2	41.05	.47
1.0504	8.3	3.252	.653
1.0989	14.2	1.787	.646
1.1431	20.2	1.414	.799
1.2045	28.0	1.239	1.317
1.2631	35.2	1.239	1.259
1.3163	41.5	1.347	1.410
1.3547	46.0	1.487	1.674
1.3994	50.4	1.672	1.582
1.4482	55.2	1.962	1.417
1.5026	60.3	2.412	1.794

TABLE XIX.

Internal Resistances (Approximate) of Batteries.

ELEMENT.	TYPE.	RESISTANCE IN OHMS.
Daniell	Callaud	4.00 to 5.00
Daniell	Meidinger	4.00 to 9.00
Grove	Ordinary	.26 to .45
Bunsen	"	.06 to .24
Grenet	"	.75 to 1.00
Leclanché	"	5.50 to 6.00
Latimer-Clark	Beetz	15700.00

TABLE XX.

Electromotive Forces of Batteries.

ELEMENT.	REMARKS.	VOLTS.
Daniell	{ Amalgamated zinc, 1 sulphuric acid + 4 water, Saturated solution of copper sulphate, Copper. }	1.07
Daniell	{ Amalgamated zinc, 1 sulphuric acid + 12 water, Saturated solution of copper sulphate, Copper. }	.97
Grove	{ Amalgamated zinc, 1 sulphuric acid + 4 water, Fuming nitric acid, Platinum. }	1.95
Bunsen	{ Amalgamated zinc, 1 sulphuric acid + 12 water, Fuming nitric acid, Carbon. }	1.94
Grenet	When freshly set up,	2.03
Leclanché	{ Amalgamated zinc, Solution of sal-ammoniac, Binoxide of manganese and carbon. }	1.46
Volta	{ Zinc, Ordinary water, Copper. }	.98
Latimer-Clark	{ Zinc, Sulphate of zinc, Sulphate of mercury in paste, Mercury. }	1.434

TABLE XXI.

Electro-chemical Equivalents.

ELEMENTS.	ATOMIC WEIGHT.	VALENCY.	CHEMICAL EQUIVALENTS.	ELECTRO-CHEMICAL EQUIVALENTS, OR GRAMS PER COULOMB OF ELECTRICITY.
Electro-positive.				
Hydrogen	1	1	1	.00001038
Potassium	39.03	1	39.03	.0004051
Sodium	23.	1	23.	.0002387
Gold	196.2	3	65.4	.0006789
Silver	107.7	1	107.7	.001118
Copper (cupric) . .	63.18	2	31.59	.0003279
Copper (cuprous) .	63.18	1	63.18	.0006558
Mercury (mercuric) .	199.8	2	99.9	.001037
Mercury (mercurous)	199.8	1	199.8	.002074
Tin (stannic) . . .	117.4	4	29.35	.0003046
Tin (stannous) . .	117.4	2	58.7	.0006093
Iron (ferric) . . .	55.88	3	18.63	.0001934
Iron (ferrous) . . .	55.88	2	27.94	.0002900
Nickel	58.6	2	29.3	.0003042
Zinc	64.88	2	32.44	.0003367
Lead	206.4	2	103.2	.001071
Aluminium	27.04	3	9.01	.0000935
Electro-negative.				
Oxygen	15.96	2	7.98	.00008283
Chlorine	35.37	1	35.37	.0003671
Iodine	126.54	1	126.54	.0013134
Bromine	79.76	1	79.76	.0008279
Nitrogen	14.01	3	4.67	.00004847

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